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RELIABILITY DISTRIBUTION OF AN INDUSTRIAL PROCESS

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ABSTARCT:
Most prominent approach used by researchers to study the behavior of the process industry systems, is through stochastic process. However it has been found that implications of stochastic process for system performance were based on exponential distribution and further behavior analysis of process industry is carried on the basis of steady state availability analysis and reliability analysis. In this paper, we formulate a stochastic model of an industrial process with an exponential distribution in order to check whether the results obtained actually exponential in nature. Results thus obtained for reliability of the process industry were analyzed for distribution fit using Minitab Software. It was found that gamma distribution fits best instead of exponential to the resulting performance data of industrial process reliability. Besides these we also checked fitting of other distributions including Normal and Weibull distribution for industrial process reliability.

Keywords: *Reliability; Industry; Probability distribution.*

1. INTRODUCTION

Various methods have been used in the field of reliability engineering to analyse the performance of industrial systems (such as Decomposition method (Barlow and Proschan, 1995), Fault Tree Analysis (Singer (1990), Misra and Weber (1989)), Bayesian network (Simon *et al.*(2008)), and stochastic process (El-Said and Agina, (2005), Epstein and Weissman (2008), Garg *et al.*(2010), Gupta *et al.* (2005, 2007), Lal *et al.*(2013) , Gupta *et al.*(2007), Knegtering and Brombacher (1999), Sachdeva *et al.*(2008, 2009), Shakuntla *et al.* (2011), Sharma and Kumar (2010), Sharma and Taneja(2011)). Most prominent approach used by researchers to study the behaviour of the process industry and its systems is through stochastic process. Stochastic process is a study of random behaviour with respect to time. These processes are extended on the basis of some assumptions to Renewal process Osaki (1970), Regenerative technique ((El-Said and Agina, (2005), Rizwan *et al.* (2010),Tuteja and Taneja (1992),Tuteja and Malik (1992), Sharma and Taneja(2011))), Markov models (Epstein and Weissman (2008), Garg *et al.*(2010), Gupta *et al.* (2005a, 2007a), Lal *et al.*(2013) , Gupta *et al.*(2007), Knegtering and Brombacher (1999), Sachdeva *et al.*(2008, 2009), Shakuntla *et al.* (2011), Kumar and Pandey (1993), Perman *et al.* (1997), Garg *et al.* (2010)) in order to study performance of industrial systems.

Some of the case studies based on steady state analysis of Markov models for improving the performance of industrial process can be found in the works of Gupta and Tiwari (2009), Kumar and Pandey (1993), Gupta *et al.* (2005b), Gupta *et al.* (2007b), Garg *et al.* (2010), Sharma *et al.*(2009), Sharma and Kumar (2008), Sharma and Kumar (2010). Narahari and Viswanadham (1994) discussed the transient analysis of manufacturing systems performance. Gupta *et al.*(2007a, 2005a) studied the system behavior under transient state numerically using Runge Kutta fourth order method for plastic pipe manufacturing plant, butter oil processing plant assuming systems have constant failure and repair rates. Lal *et al.* (2013) also studied the transient behaviour of piston manufacturing plant. Kaur *et al.* (2013) proposed a numerical method to study the transient analysis for a system having variable repair rate and fixed failure rate. It has been found that implications of stochastic process for system performance were based on exponential distribution and further behavior analysis of process industry is carried on the basis of steady state availability analysis and reliability

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analysis. In this paper, we formulate a stochastic model for an industrial process with an exponential distribution in order to check whether the results obtained actually exponential in nature. Before we proceed some of the preliminary concepts are discussed in next section.

2. PRILIMINARY CONCEPTS

British Standards Institution as given in (Dummer and Winton, 1990) stated reliability, as the characteristic of an item expressed by the probability that it will perform a required function under stated conditions for a stated period of time. Probabilistically, it can be interpreted as,

$$R(t) = P[T > t]\,,\, t \geq 0 \tag{1}$$

where T is the random variable representing failure time or time to failure.

From this, the unreliability or the failure probability distribution function, $F(t)$ can be defined as

$$F(t) = 1 - R(t) = P[T \leq t] \tag{2}$$

The probability distribution function in terms of probability density function, $f(t)$ is given as follows:

$$F(t) = \int_{-\infty}^{T} f(t)\,dt \tag{3}$$

The following failure distribution of the system or process is discussed in this paper.

2.1. *Exponential distribution*

A continuous random variable having range $0 \leq t \leq \infty$ is said to have exponential distribution if it has the probability density function, $f(t)$ is of the following form,

$$f(t) = \begin{cases} \lambda e^{-\lambda t} & 0 < t < \infty \\ 0, & t < 0 \end{cases} \tag{4}$$

2.2. *Weibull distribution*

A continuous random variable is said to have Weibull distribution if it has the probability density function, is of the following form,

$$f(t) = at^{b} e^{-(\frac{at^{b+1}}{b+1})}\,,\, t \geq 0 \tag{5}$$

where a and b are positive constant and are known as scale and shape parameters, respectively.

2.3. *Gamma distribution*

If the continuous random variable has the probability density function is of the following form, then t is said to have Gamma distribution.

$$f(t) = ct^{a-1}e^{-bt}\,,\, t \geq 0 \tag{6}$$

where a and b are positive constant and c can be obtained in terms of a and b, and from the following equation

$$c\int_{0}^{\infty} t^{a-1}e^{-bt}\,dt = 1 \tag{7}$$

2.4. *Normal distribution*

If the continuous random variable has the probability density function is of the form, it is said to have Normal distribution:

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}}, -\infty \le t \le \infty \tag{8}$$

where the constant μ and σ are arbitrary and represent the mean and standard deviation of the random variable.

3. INDUSTRIAL PROCESS AND ITS RELIABILITY FORMULATION

A fabric industry is selected in order to check whether the reliability of the process industry actually exponential in nature. We divide the industrial process into six sub-systems and the description of each sub-system function, number of machines, their failure causes and Mean time to failure (MTTF) of each sub-system can be referred from Table 1.

Now as per table description, we have industrial process as a complex system of six subsystems to woven a fabric and termed as A, B, C, D, E, F. Sub-system A as storage system considered to be never failing due to well-established environmental conditions. Sub-system B (Warper) have three machines and process shows failures if two of its machines fails i.e. process fails to achieve set target production of fabric due to simultaneous failure of these machines as one of the production line stop giving required output. Similarly, Sub-system C, D, E and F have one, three, two hundred, and three machines respectively and failure of one, two, twenty and two machines leads to process failure.

Table I

Sub-system	**Function**	**No. of machines**	**Assumptions***	**Sub-system MTBF (in hrs)**
Storage Room (A)	Stores the raw material	3	Never fails	100000
Warper (B)	Prepare base for fabric	3	Failed when two warper machines fails out of three	72000
Mixture (C)	For coating to give strength to thread	1	Failed when shows any failure	21000
Slasher (D)	Coating of mixture and drying the threads after coating	3	Failed when two out of three machines fails	15000
Loom(E)	It interlaces threads to woven fabric as final product.	200	Failed when 20 out of 200 fails	54000
Fabric Inspection Machine (F)	To check the final product quality	2	Failed when both machines failed together	75000
*Initially all sub-systems are operating. The failure of each sub-system is independent of other sub-systems. Each Sub-system has optimised number of skilled workers with necessary spare inventory.				

The notations used under assumptions (Table I) of the process are discussed as:

i. Failure rate of the sub-systems represented by $\lambda_{i,}$, $i = 1...5$ respectively for sub-system B, C, D, E, F.

ii. Probability that system is in $i\,th$ state at time t, is denoted by $P_i(t)$, for $i = 1, 2, ..., 5$.

iii. The rate of change of state with respect to time t at the ith state of the system is represented by $P_i'(t)$, for $i = 1, 2, ..., 5$.

The reliability of the process using stochastic model computed as

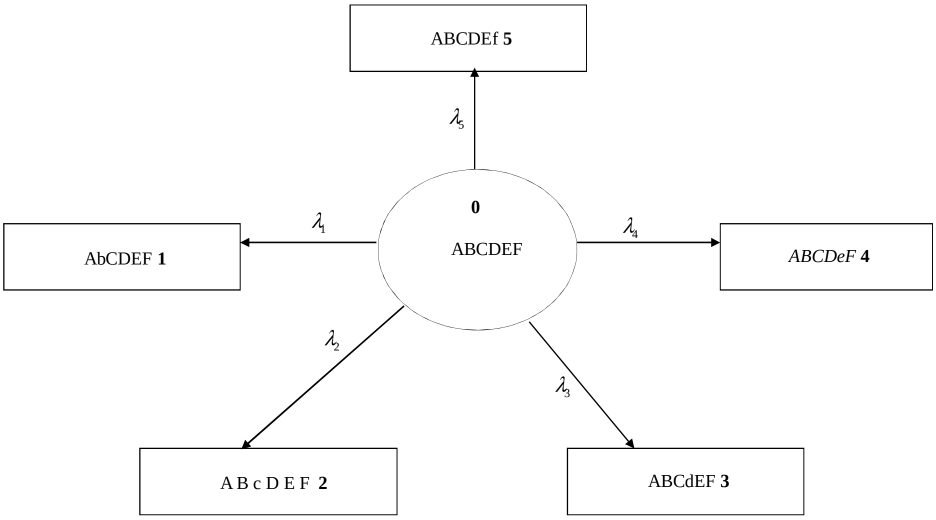
$$R_1(t) = 1 - \sum_{i=1}^{5} P_i(t) = 0 \tag{9}$$

The values of transient probabilities in the above equations thus, obtained by solving following equations (10-11) simultaneously using Runge-Kutta Method as discussed in Gupta *et al.* (2005, 2007), Lal *et al.*(2013).

$$P_0'(t) + \sum_{i=1}^{5} \lambda_i P_0(t) = 0, \tag{10}$$

$$P_i'(t) = \lambda_i P_0(t), \quad for\, i = 1, 2, 3, 4, 5. \tag{11}$$

The equations (10-11) were obtained following a mnemonic rule discussed in [] on a state space diagram, given in Fig 1. In the next section, we will discuss the results obtained in order to check the distribution of reliability of the process industry.



In this figure, the operating conditions of sub-systems are represented by capital letters, as *A, B, C, D, E* and *F* while lowercase letters, including *a, b, c, d, c e,* and *f* represent the failed states of the modules *A, B, C, D, E* and *F,* respectively. The failed states of the process are represented by rectangles and are numbered from one to five. The operating state of the process (when all modules are working satisfactorily) is denoted by a zero and symbolized by a circle. The constant failure rates of the subsystems are represented by λ_i and $i = 1,2,3,4,5$, respectively, for modules *B, C, D, E,* and *F*.

Figure 1: State space diagram of industrial process

4. RESULTS AND DISCUSSION

We solved the equations (10 and 11) for evaluating the reliability of the process industry from Equation (9). Then we check distribution fit on the resulting data through probability plots using MINITAB software. Probability plots include percentiles points for corresponding probabilities of an ordered data set. The middle line is the expected percentile from the distribution based on maximum likelihood parameter estimates. The left and right line represents the lower and upper bounds for the confidence intervals of each percentile. The probability plots are shown in Figure 2 for four types of distributions fit on reliability of the process industry. These distributions are exponential, Weibull, Normal and gamma distribution.

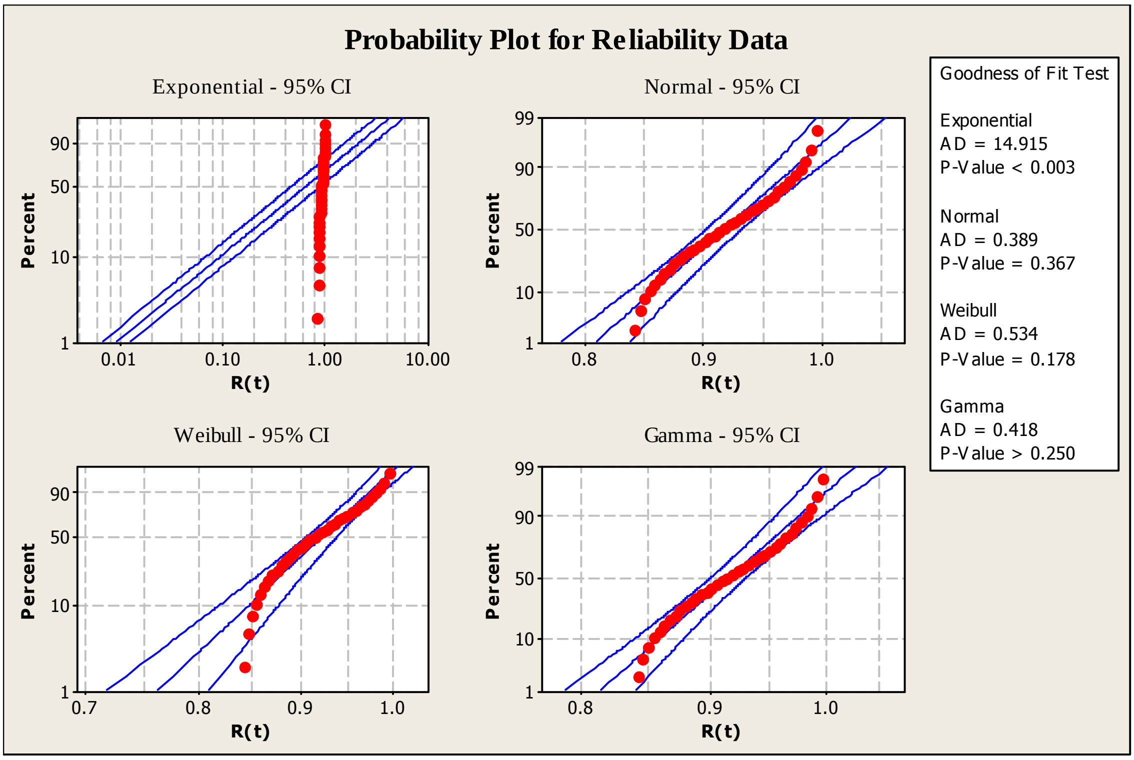


Figure 2: Probability plots fitting on Reliability Data for Exponential, Weibull, Normal and Gamma Distributions

If the p value is greater than a given value this implies the best fit distribution. The resulting plots (Figure 2) suggests that Gamma distribution is best fit while the Normal and Weibull shows equally good fit in comparison to Exponential distribution. Here we conclude that in order to define the reliability of an industry, the gamma distribution can be beneficiary than the other three distributions as the resulting analysis of stochastic process obtained for its state probabilities are not exponential in nature.

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