Theoretical and Numerical Analysis of Vibration Isolator Subjected to Harmonic Excitation

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Abstract- Vibration isolation is a method of adding a tuned spring-mass damper to a system to decrease the force transmitted from the vibrating machine to the foundation or vice versa. The dynamic vibration isolator is designed in such a way that the force transmitted from the exciter to the foundation should be minimized. In this paper theoretical and numerical analysis of vibration isolator subjected to harmonic excitation is carried out. Proposed experimental setup for dynamic vibration isolator is also designed.

Index Terms- Dynamic, Exciter, Harmonic Excitation, Vibration Isolator

1. INTRODUCTION:

The purpose of vibration isolation is to control unwanted vibrations so that the adverse effects are kept within acceptable limits. Transmissibility is a concept widely used in the design of vibration isolators to measure the vibration transmission at different frequencies [1].

Vibration isolation concerns means to bring about a reduction in a vibratory effect. A vibration isolator in its most elementary form may be considered as a resilient member connecting the equipment and foundation. The function of an isolator is to reduce the magnitude of motion transmitted from a vibrating foundation to the equipment or to reduce the magnitude of force transmitted from the equipment to its foundation [6].

Inserting the vibration isolator between the source of vibration and the vibration receiver is one of the fundamental ways to reduce the unwanted vibrations and to protect the equipment's from disturbance. The basic concept of the vibration isolator is that, when the frequency of excitation is larger than $\sqrt{2}\omega_n$, where ω_n is the undamped natural frequency of the isolator, the transmitted force, Ft (or the transmitted displacement, Xt) reaches a value less than the excitation force, Fi (or the excitation displacement, Xi). The ratio Ft/Fi and Xt/Xi are denoted as force transmissibility and displacement transmissibility respectively [2].

In the design and analysis of vibration isolator the damping coefficient and spring stiffness of isolator plays important role. In this paper the effect of the damping coefficient and spring stiffness on force and displacement transmissibility will be done and verification of the same will be done through MATLAB. For this analysis vibration isolator system is excited by harmonic Excitation.

2. THEORETICAL BACKGROUND:

Mathematical model of vibration isolator is as shown in Fig. 2.1 below [3].

Where,

$m =$ Mass of system

$k =$ Stiffness of the spring

$c =$ Damping coefficient

Force Transmissibility: The force transmissibility is defined as the ratio of the force transmitted to the foundation to that impressed upon the system [3].

It is assumed that the operation of the machine gives rise to a harmonically varying force.

$$F(t) = F_0 \sin\omega t \quad (1)$$

The equation of motion of the machine (of mass m) is given by

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t \quad (2)$$

Since the transient solution dies out after some time, only the steady-state solution will be left. The steady-state solution of Eq. (2) is given by

$$x(t) = X\sin(\omega t - \phi) \quad (3)$$

where

$$X = \frac{F_0}{[(k-m\omega^2)^2 + \omega^2 c^2]^{1/2}} \quad (4)$$

and

$$\emptyset = \tan^{-1}\left(\frac{\omega c}{k - m\omega^2}\right)$$

The force transmitted to the foundation through the spring and the dashpot, is given by

$$F_t(t) = kx(t) + c\dot{x}(t)$$
$$= kX\cos(\omega t - \emptyset) - c\omega X\sin(\omega t - \emptyset) \quad (5)$$

The magnitude of the total transmitted force (F_t) is given by

$$F_t = [(kx)^2 + (c\dot{x})^2]^{1/2} = X\,(k^2 + \omega^2 c^2)^{1/2}$$

$$= \frac{F_i\,(k^2 + \omega^2 c^2)^{1/2}}{[(k - m\omega^2)^2 + \omega^2 c^2]^{1/2}} \quad (6)$$

The transmissibility or transmission ratio of the isolator (T_f) is defined as the ratio of the magnitude of the force transmitted to that of the exciting force:

$$(T_f) = \frac{F_t}{F_i} = \{\frac{k^2 + \omega^2 c^2}{(k - m\omega^2)^2 + \omega^2 c^2}\}^{1/2}$$

$$= \{\frac{1 + (2\zeta r)^2}{[1 - r^2]^2 + (2\zeta r)^2}\}^{1/2} \quad (7)$$

Where,

$r = \frac{\omega}{\omega_n}$ is the frequency ratio.

From equation (7) it is clear that when $r = 0$, i.e. system frequency is zero the $(T_f) = 1$. For various values of damping coefficient, force transmissibility versus frequency ratio curve is shown in Fig. 2.2.

The following observations can be made from Fig. 2.2

1. The magnitude of the force transmitted to the foundation can be reduced by decreasing the natural frequency of the system (ω_n)
2. The force transmitted to the foundation can also be reduced by decreasing the damping ratio. However, since vibration isolation requires $r > (2)^{1/2}$ the machine should pass through resonance during start-up and stopping. Hence, some damping is essential to avoid infinitely large amplitudes at resonance.
3. Although damping reduces the amplitude of the mass (X) for all frequencies, it reduces the maximum force transmitted to the foundation F_t only if $r < (2)^{1/2}$. Above that value, the addition of damping increases the force transmitted.

Displacement Transmissibility: The displacement transmissibility is defined as the ratio of absolute amplitude of the mass to the base excitation amplitude [4].

In many applications, the isolation is required to reduce the motion of the mass (machine) under the applied force. The displacement amplitude of the mass m due to the force F(t), given by Eq. (4), can be expressed as:

$$T_d = \frac{X}{\delta_{st}} = \frac{kX}{F_0} = \frac{1}{([1 - r^2]^2 + (2\zeta r)^2)^{1/2}} \quad (8)$$

Where $\frac{X}{\delta_{st}}$ is called, in the present context, the displacement transmissibility or amplitude ratio and indicates the ratio of the amplitude of the mass, X, to the static deflection under the constant force F_0, $\delta_{st} = \frac{F_0}{k}$. The variation of the displacement transmissibility with the frequency ratio r for several values of the damping ratio ζ is shown in Fig. 2.3.

The following observations can be made from Fig. 2.3

1) The displacement transmissibility increases to a maximum value at

$$r = (1 - 2\zeta^2)^{1/2} \quad (9)$$

Equation (8) shows that, for small values of damping ratio ζ the displacement transmissibility will be maximum at $r \approx 1$ or $\omega \approx \omega_n$. Thus the value of r is to be avoided in practice. In most cases, the excitation frequency is fixed and hence we can avoid by altering the value of the natural frequency which can be accomplished by changing the value of either or both of m and k.

2) The amplitude of the mass, X, approaches zero as r increases to a large value. The reason is that at large values of r, the applied force $F(t)$ varies very rapidly and the inertia of the mass prevents it from following the fluctuating force.

Table 2.1. Various Parameters of the Vibration Isolator System.

Parameter	Value
m	10 kg
k	5510 N/m
c	66.39 N-s/m
ω_n	33.196 rad/s
ζ	1.2

3. NUMERICAL ANALYSIS:

In order to carry out numerical analysis of vibration isolator MATLAB software is used. MATLAB integrates mathematical computing, visualization, and a powerful language to provide a flexible environment for technical computing. The open architecture makes it easy to use MATLAB and its companion products to explore data, create algorithms, and create custom tools that provide early insights and competitive advantages. MATLAB program has been developed to model the vibration isolator system [5].

Using MATLAB, Transmissibility of vibration isolator for Eq. (7) as functions of the frequency ratio is plotted as shown in Fig 3.1

Using MATLAB, Displacement Transmissibility of vibration isolator for Eq. (8) as functions of the frequency ratio is plotted as shown in Fig 3.2

4. EXPERIMENTAL SETUP

Proposed experimental setup for vibration isolator is developed by considering the parameters given in the table. Base structure is designed to support the vibration isolator system. The transmissibility curves of vibration isolator are plotted using FFT analyzer. The result obtained will be real because in theoretical and numerical analysis effect of nonlinearities is not considered, and the effect of nonlinearities on transmissibility will be evaluated by experimental analysis.

5. CONCLUSION:

In this paper theoretical and numerical analysis of design of vibration isolator is carried out. Proposed experimental setup for vibration isolator is also designed. Transmissibility curves obtained from numerical analysis and theoretical analysis are matching. The analysis can be carried out for vibration isolator subjected to random excitation. By carrying experimental analysis for vibration isolator subjected to random excitation the result are validated with theoretical and numerical analysis.

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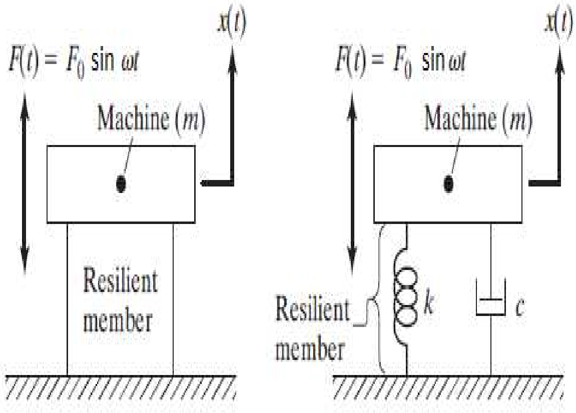

Fig.2.1. Vibration Isolator

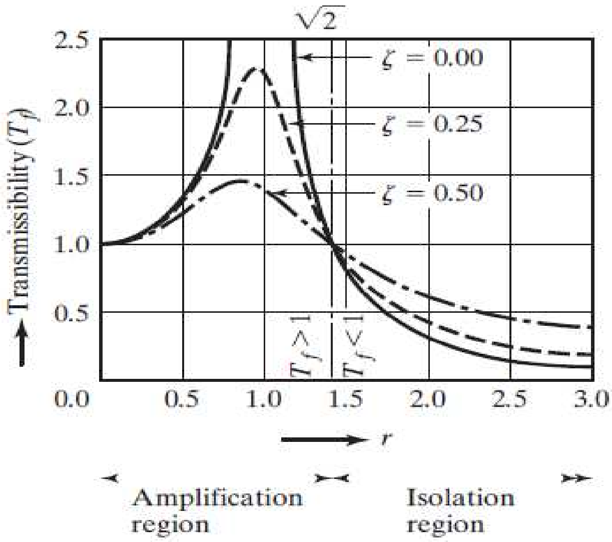

Fig 2.2 Force Transmissibility (T_f) Vs Frequency Ratio (r) of Theoretical Analysis

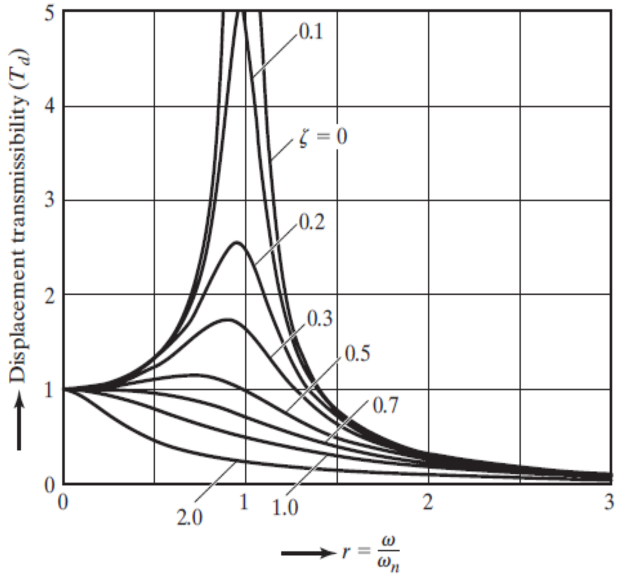

Fig 2.3 Displacement Transmissibility (T_d) Vs Frequency Ratio $r = \frac{\omega}{\omega_n}$ of Theoretical Analysis

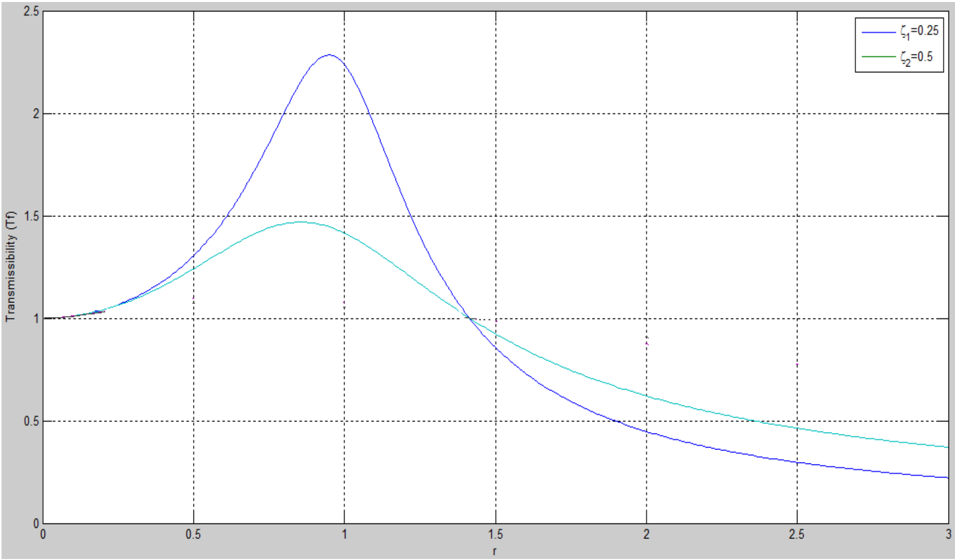

Fig 3.1 Force Transmissibility (T_f) Vs Frequency Ratio (r) of Numerical Analysis

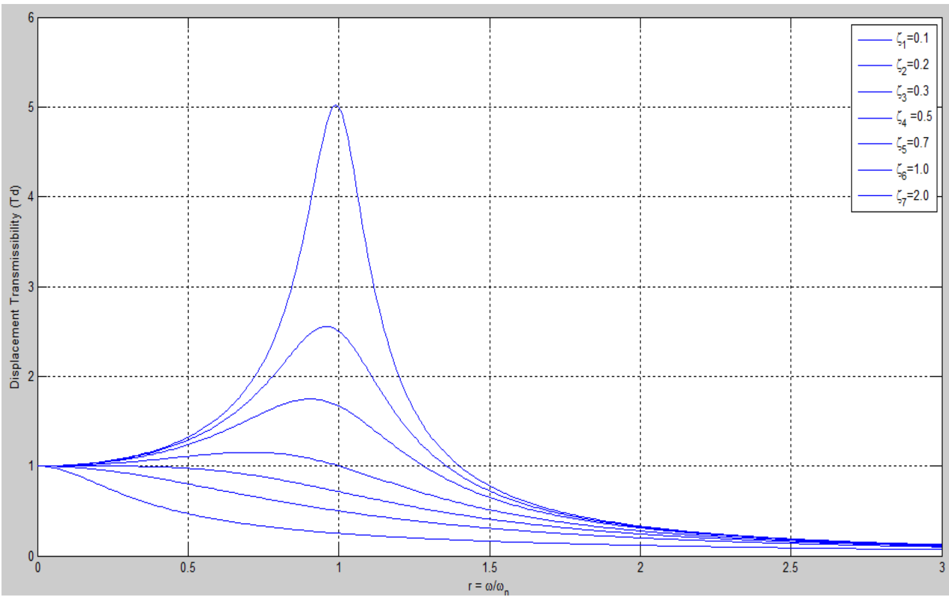

Fig 3.2 Displacement Transmissibility (T_d) Vs Frequency Ratio $(r = \frac{\omega}{\omega_n})$ of Numerical Analysis

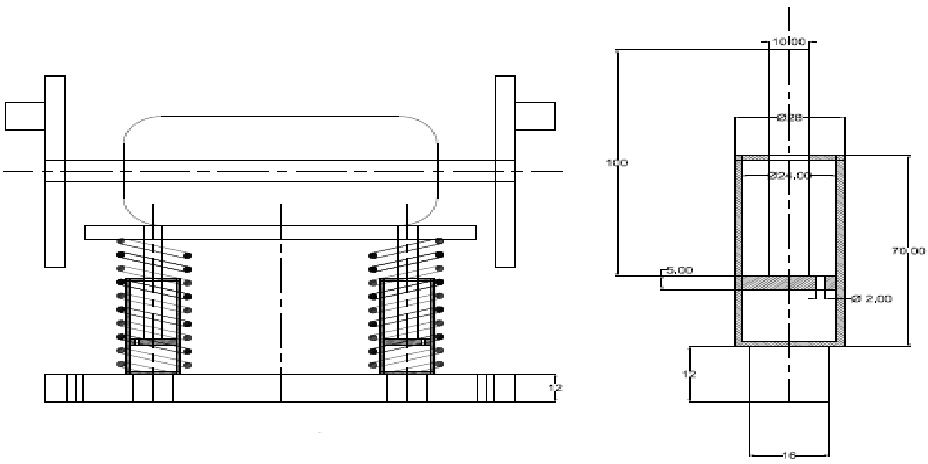

Fig.4.1 Proposed Experimental Setup of Vibration Isolator