A Technique to Solve Parametric Type Triangular Number Fuzzy Transportation Problem

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Abstract-This paper discusses an optimization technique to solve Fuzzy Transportation Problem. Initially the cost, demand, supply and allocation in cell are considered as triangular fuzzy numbers. Then those fuzzy numbers are changed into parametric form. Finally this parameterized fuzzy transportation problem is solved, so as to find the Optimum solution.

Keywords-Triangular fuzzy number; Parametric form; IBFS; Optimum solution.

1. INTRODUCTION

Bellman and Zadeh[1] introduced the concept of fuzzy set theory in decision making. In traditional Mathematical formulation of Transportation problem all the parameters are treated as deterministic in nature. However, in real life, uncertainty always exists. Due to this vagueness the concept of fuzzy set theory introduced by zadeh[7] is more suitable in Transportation problem. There are many research articles on fuzzy Transportation problem [4],[6]. Malini et.al [2] used ranking technique to solve trapezoidal number fuzzy Transportation problem. In 2011 Sagaya Roseline et.al [5] proposed a generalized fuzzy modified distribution method for generalized fuzzy transportation problem.

This paper discusses an optimization technique to solve parametric type Triangular number fuzzy Transportation problem. Here, the triangular fuzzy numbers are taken in parametric form. For this, parametric type Fuzzy Transportation problem, the initial basic feasible solution(IBFS) and the optimum solution are found using the proposed method. The model is illustrated with an example.

2. PRELIMINARIES

2.1 *Definition*

A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0,1]. (i.e) A= {(x,μ_A(x)/xϵX}, here μ_A:X→ [0,1] is a mapping called the degree of membership function of the fuzzy set A and μ_A is called the membership value of xϵX in the fuzzy set A.

2.2 *Definition*

It is a fuzzy number, if it satisfies the following:

- A_α is convex (i.e. interval)∀ $\alpha \in$ [0,1]
- Normal (there exists x_0 ∈R with $\mu_A(x_0)$ =1
- A_α is closed interval (with the end points)∀α ∈(0,i)

Then $\tilde{A}$ is called fuzzy interval. Moreover if there exists only one $x_0 \in$ R with $\mu_A(x_0)$ = 1 then $\widetilde{A}$ is called fuzzy number.

2.3 *Definition*

Among the various types of fuzzy number, triangular fuzzy number(TFN) is the most popular one. If A is a fuzzy number represented with three points as follows. The triangular fuzzy number A,
can be represented by A(a_1,a_2,a_3;1) with membership function μ(X) given by

$$\mu_A(\mathrm{x})=\begin{cases} 0 & ; \quad x < \mathrm{a}_1 \\ \frac{\mathrm{x}-\mathrm{a}_1}{\mathrm{a}_2-\mathrm{a}_1} & ;\ \mathrm{a}_1 \le \mathrm{x} \le \mathrm{a}_2 \\ \frac{\mathrm{a}_3-\mathrm{x}}{\mathrm{a}_3-\mathrm{a}_2} & ;\ \mathrm{a}_2 \le \mathrm{x} \le \mathrm{a}_3 \\ 0 & ; \quad \mathrm{x} > \mathrm{a}_1 \end{cases}$$

2.4 *Parametric type Triangular Fuzzy Number*

Let $\tilde{\alpha}$=$(\alpha_1, \alpha_2, \alpha_3)$ be a triangular fuzzy number then the parametric form of the TFN is defined as $\tilde{\alpha}$=$(\alpha_0, \alpha_*, \alpha^*)$ where $\alpha_* = \alpha_0 - \underline{a}$ and $\alpha^* = \overline{a} - \alpha_0$, $\overline{a}(p)$=$\alpha_3 - (\alpha_3 - \alpha_2)p$ and $\underline{a}(p) = (\alpha_2 - \alpha_1)p + \alpha_1$, $\alpha_0 = \frac{\underline{a}(p)+\overline{a}(p)}{2}$ when p=1, we get $\alpha_0 = \alpha_2$ where p∈[0,1]

2.5 *Arithmetic Operations on Parametric type Triangular Fuzzy Number*

Ming Ma et al.[3] have proposed a new fuzzy arithmetic operation based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice L. That is for $\alpha,\beta \in$L we define $\alpha \vee \beta$ =max{ α, β} and $\alpha \wedge \beta$=min{ α, β}

For arbitrary triangular fuzzy numbers $\tilde{\alpha} = (\alpha_0, \alpha_*, \alpha^*)$ $\tilde{\beta} = (\beta_0, \beta_*, \beta^*)$ and *={+,–,×,÷}, the arithmetic operations are defined as

$\tilde{\alpha} * \tilde{\beta} = (\alpha_0, \alpha_*, \alpha^*) * (\beta_0, \beta_*, \beta^*)$=$(\alpha_0 * \beta_0, \alpha_* * \beta_*, \alpha^* * \beta^*)$

= $(\alpha_0 * \beta_0$,max$\{\alpha_* * \beta_*\}$,max$\{\alpha^* * \beta^*\}$

In particular for any two triangular fuzzy numbers $\tilde{\alpha} = (\alpha_0, \alpha_*, \alpha^*)$ and $\tilde{\beta} = (\beta_0, \beta_*, \beta^*)$, we define

2.5.1 Addition: $\tilde{\alpha} + \tilde{\beta} = (\alpha_0, \alpha_*, \alpha^*) + (\beta_0, \beta_*, \beta^*)$

= $(\alpha_0 + \beta_0$,max$\{\alpha_*, \beta_*\}$, max$\{\alpha^*, \beta^*\}$

2.5.2 Subtraction: $\tilde{\alpha} - \tilde{\beta} = (\alpha_0, \alpha_*, \alpha^*) - (\beta_0, \beta_*, \beta^*)$

= $(\alpha_0 - \beta_0$,max$\{\alpha_*, \beta_*\}$,max$\{\alpha^*, \beta^*\})$

2.5.3 Multiplication: $\tilde{\alpha} \times \tilde{\beta} = (\alpha_0, \alpha_*, \alpha^*) \times (\beta_0, \beta_*, \beta^*)$

= $(\alpha_0 \times \beta_0$,max$\{\alpha_*, \beta_*\}$,max$\{\alpha^*, \beta^*\})$

2.5.4 Division: $\tilde{\alpha} \div \tilde{\beta} = (\alpha_0, \alpha_*, \alpha^*) \div (\beta_0, \beta_*, \beta^*)$

= $(\alpha_0 \div \beta_0$,max$\{\alpha_*, \beta_*\}$,max$\{\alpha^*, \beta^*\})$

3. ALGORITHM TO SOLVE PARAMETRIC TYPE FUZZY TRANSPORTATION PROBLEM

3.1 *Mathematical Formulation of Fuzzy Transportation Problem*

Consider a fuzzy transportation with m sources n destinations with triangular fuzzy numbers. Let$\tilde{a_i} \geq \tilde{o}$ be the fuzzy availability at source i and $\tilde{b_j}$, ($\tilde{b_j} \geq \tilde{o}$) be the requirement at destination j. Let $\tilde{C}_{ij}$ ($\tilde{C}_{ij} \geq \tilde{o}$)be the unit fuzzy transportation cost from source i to destination j. Let $\tilde{X}_{ij}$ denote the number of fuzzy units to be transformed from source i to destination j. Now the problem is to find a feasible way of transporting the available amount at each source to satisfy the demand at each destination so that the total transportation cost is minimized.

The mathematical model of fuzzy transportation problem is as follows

Minimize $\tilde{Z} \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij} \tilde{X}_{ij}$

Subject to $\sum_{j=1}^{n} \tilde{X}_{ij} \approx \tilde{a}_i$, i =1,2,3….m

$\sum_{i=1}^{m} \tilde{X}_{ij} \approx \tilde{b}_j$, j =1,2,3….n ----------------(1)

$\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j}^{n} \tilde{b}_j$, i =1,2,3…..m;

j = 1,2,3….n and $\tilde{X}_{ij} \geq \tilde{0}$ for all i and j.

Where $\tilde{C}_{ij}$ is the fuzzy unit transportation cost from ith source to the jth destination

3.2 *Steps to find IBFS to Parametric type Fuzzy Transportation Problem*

Here, IBFS is found using PARAMETRIC TYPE LEAST COST method.

Step 1: Consider the Parametric type fuzzy transportation problem with triangular fuzzy demand and

triangular fuzzy supply.

Step 2: Convert all the triangular fuzzy supply and fuzzy demand into parametric form of Triangular Fuzzy Number,

$\tilde{\alpha}$=$(\alpha_0, \alpha_*, \alpha^*)$ where $\alpha_* = \alpha_0 - \underline{a}$ and $\alpha^* = \overline{a} - \alpha_0$, $\overline{a}(p)$=$\alpha_3 - (\alpha_3 - \alpha_2)p$,

$\underline{a}(p) = (\alpha_2 - \alpha_1)p + \alpha_1$,$\alpha_0 = \frac{\underline{a}(p)+\overline{a}(p)}{2}$ here p=1, we get $\alpha_0 = \alpha_2$

Step 3:Similarly convert all the fuzzy cost co-efficient into the same Parametric type.

Step 4: Determine the smallest cost in the cost matrix of the transportation table. Let it be $\widetilde{c_{ij}}$, Allocate

$\widetilde{x_{ij}}$= min ($\tilde{a_i}$,$\tilde{b_j}$) in the cell (i,j)

Step 5: If $\widetilde{x_{ij}}$=$\tilde{a_i}$ cross off the ith row of the transportation table and decrease $\tilde{b_j}$ by $\tilde{a_i}$. Go to step 3. If $\widetilde{x_{ij}}$= $\tilde{b_j}$

cross off the j th column of the transportation table and decrease $\tilde{a_i}$ by $\tilde{b_j}$. Go to step 3.

If $\widetilde{x_{ij}}$=$\tilde{a_i}$=$\tilde{b_j}$ cross off either the ith row or jth column but not both.

step 6: Repeat steps 3 and 4 for the resulting reduced transportation table until all the rim requirements are satisfied.

Whenever the minimum cost is not unique, make an arbitrary choice among the minima.

3.3 *Optimum Solution to Parametric type of Fuzzy Transportation Problem*

Here, we have used PARAMETRIC TYPE MODI Method to find the Optimum Solution to the Parametric type Fuzzy Transportation Problem. The arithmetic operations proposed in section 2.5 are used to find the optimum solution to the parametric type triangular number fuzzy transportation problem.

4. NUMERICAL EXAMPLE

Parametric form technique to solve fuzzy transportation problem

Solve the following Fuzzy Transportation Problem using Parametric form of Triangular Fuzzy Number.

	D1	D2	D3	D4	Supply
S1	(0,1,2)	(1,2,4)	(0,1,4)	(3,4,7)	(25,30,40)
S2	(1,3,6)	(1,3,4)	(0,2,5)	(0,1,3)	(20,30,50)
S3	(1,4,8)	(1,2,5)	(2,5,9)	(6,9,15)	(15,20,25)
Demand	(10,20,25)	(20,40,50)	(20,30,40)	(5,10,20)	

Solution:

Here each allocation, Demand and Supply are Fuzzy Triangular Number.

Parametric form of Triangular Fuzzy Number

$\tilde{\alpha}=(\alpha_0,\alpha_*,\alpha^*)$ where $\alpha_* = \alpha_0 - \underline{a}$ and $\alpha^* = \overline{a} - \alpha_0$, $\overline{a}(p)=\alpha_3 - (\alpha_3 - \alpha_2)p$, $\underline{a}(p) = (\alpha_2 - \alpha_1)p + \alpha_1$, $\alpha_0 = \frac{\underline{a}(p)+\overline{a}(p)}{2}$ here p=1, we get $\alpha_0 = \alpha_2$

The parametric form of all triangular fuzzy numbers in the fuzzy transportation table are,

Table 1

	D1	D2	D3	D4	Supply
S1	(1,1-p,1-p)	(2,1-p,2-2p)	(1,1-p,3-3p)	(4,1-p,3-3p)	(30,5-5p,10-10p)
S2	(3,2-2p,3-3p)	(3,2-2p,1-p)	(2,2-2p,3-3p)	(1,1-p,2-2p)	(30,10-10p,20-20p)
S3	(4,3-3p,4-4p)	(2,1-p,3-3p)	(5,3-3p,4-4p)	(9,3-3p,6-6p)	(20,5-5p,5-5p)
Demand	(20,10-10p,5-5p)	(40,20-20p,10-10p)	(30,10-10p,10-10p)	(10,5-5p,10-10p)	

Table 2: Finding IBFS using Parametric type Least cost method

	D1	D2	D3	D4	Supply
S1	(1,1-p,1-p) **(10,10-10p,20-20p)**	(2,1-p,2-2p)	(1,1-p,3-3p) **(20,10-10p,20-20p)**	(4,1-p,3-3p)	(30,5-5p,10-10p)
S2	(3,2-2p,3-3p)	(3,2-2p,1-p) **(20,10-10p,20-20p)**	(2,2-2p,3-3p) **(10,10-10p,20-20p)**	(1,1-p,2-2p)	(30,10-10p,20-20p)
S3	(4,3-3p,4-4p)	(2,1-p,3-3p) **(20,5-5p,5-5p)**	(5,3-3p,4-4p)	(9,3-3p,6-6p)	(20,5-5p,5-5p)
S4	(0,3-3p,4-4p) **(10,10-10p,20-20p)**	(0,2-2p,3-3p)	(0,3-3p,4-4p)	(0,3-3p,6-6p) **(10,5-5p,10-10p)**	(20,10-10p,20-20p)
Demand	(20,10-10p,5-5p)	(40,20-20p,10-10p)	(30,10-10p,10-10p)	(10,5-5p,10-10p)	

The IBFS in Parametric form is = (150,10-10p,20-20p)

The fuzzy IBFS for different values of p are

Table 3

P	IBFS
0	(150,10,20)
0.5	(150,5,10)
1	(150)

Table 4: Optimum Solution

	D1	D2	D3	D4	Supply
S1	(1,1-p,1-p) **(10,10-10p,20-20p)**	(2,1-p,2-2p)	(1,1-p,3-3p) **(20,10-10p,20-20p)**	(4,1-p,3-3p)	u_1=0
S2	(3,2-2p,3-3p)	(3,2-2p,1-p) **(20,10-10p,20-20p)**	(2,2-2p,3-3p) **(10,10-10p,20-20p)**	(1,1-p,2-2p)	u_2=(1,2-2p,3-3p)
S3	(4,3-3p,4-4p)	(2,1-p,3-3p) **(20,5-5p,5-5p)**	(5,3-3p,4-4p)	(9,3-3p,6-6p)	u_3=(0,2-2p,3-3p)
S4	(0,3-3p,4-4p) **(10,10-10p,20-20p)**	(0,2-2p,3-3p)	(0,3-3p,4-4p)	(0,3-3p,6-6p) **(10,5-5p,10-10p)**	u_4=(-1,3-3p,4-4p)
Demand	v_1=(1,1-p,1-p)	v_2=(2,2-2p,3-3p)	v_3=(1,1-p,3-3p)	v_4=(1,3-3p,6-6p)	

z_{12}=2+0-2=0, z_{14}=1+0-4=-3, z_{21}=1+1-3=-1, z_{24}=1+1-1=1, z_{31}=1+0-4=-3,
z_{33}=1+0-5=-4, z_{34}=1+0-9=-8, z_{42}=2-1+0=1, z_{43}=1-1-0=0.

Table 5

	D1	D2	D3	D4	Supply
S1	(1,1-p,1-p) **(0,10-10p,20-20p)**	(2,1-p,2-2p)	(1,1-p,3-3p) **(30,10-10p,20-20p)**	(4,1-p,3-3p)	u_1=(-1,2-2p,3-3p)
S2	(3,2-2p,3-3p)	(3,2-2p,1-p) **(20,10-10p,20-20p)**	(2,2-2p,3-3p) **(0,10-10p,20-20p)**	(1,1-p,2-2p) **(10,5-5p,10-10p)**	u_2=0
S3	(4,3-3p,4-4p)	(2,1-p,3-3p) **(20,5-5p,5-5p)**	(5,3-3p,4-4p)	(9,3-3p,6-6p)	u_3=(-1,2-2p,3-3p)
S4	(0,3-3p,4-4p) **(20,10-10p,20-20p)**	(0,2-2p,3-3p)	(0,3-3p,4-4p)	(0,3-3p,6-6p)	u_4=(-2,3-3p,4-4p)
Demand	v_1=(2,2-2p,3-3p)	v_2=(3,2-2p,1-p)	v_3=(2,2-2p,3-3p)	v_4=(1,1-p,2-2p)	

z_{12}=3-1-2=0, z_{14}=1-1-4=-4, z_{21}=2+0-3=-1, z_{31}=2-1-4=-3, z_{33}=2-1-5=-4,
z_{34}=1-1-9=-9, z_{42}=3-2=1, z_{43}=2-2=0, z_{44}=1-2=-1.

Table 6

	D1	D2	D3	D4	Supply
S1	(1,1-p,1-p) **(20,10-10p,20-20p)**	(2,1-p,2-2p)	(1,1-p,3-3p) **(10,10-10p,20-20p)**	(4,1-p,3-3p)	u_1=(-1,2-2p,3-3p)

S2	(3,2-2p,3-3p)	(3,2-2p,1-p) **(0,10-10p,20-20p)**	(2,2-2p,3-3p) **(20,10-10p,20-20p)**	(1,1-p,2-2p) **(10,5-5p,10-10p)**	u_2=0
S3	(4,3-3p,4-4p)	(2,1-p,3-3p) **(20,5-5p,5-5p)**	(5,3-3p,4-4p)	(9,3-3p,6-6p)	u_3=(-1,2-2p,3-3p)
S4	(0,3-3p,4-4p)	(0,2-2p,3-3p) **(20,10-10p,20-20p)**	(0,3-3p,4-4p)	(0,3-3p,6-6p)	u_4=(-3,2-2p,3-3p)
Demand	v_1=(2,2-2p,3-3p)	v_2=(3,2-2p,1-p)	v_3=(2,2-2p,3-3p)	v_4=(1,1-p,2-2p)	

z_{12}=3-1-2=0, z_{14}=1-1-4=-4, z_{21}=2+0-3=-1, z_{31}=2-1-4=-3, z_{33}=2-1-5=-4,
z_{34}=1-1-9=-9, z_{41}=2-3=-1, z_{43}=2-3=-1, z_{44}=1-3=-2.

All $z_{ij} \leq 0$ The given table is optimal.

The Optimum Transportation cost in Parametric Form = (120,10-10p,20-20p)

The fuzzy optimal solution in different values of p are

Table 7

P	OPTIMUM TRANSPORTATION COST
0	(120,10,20)
0.5	(120,5,10)
1	(120)

5. CONCLUSION

In this paper, we have presented a Transportation Problem where demand, supply and cost are considered as Triangular Fuzzy Numbers instead of crisp or probabilistic in nature to make the Transportation Problem more realistic. At first, Parametric type Triangular fuzzy numbers are developed. Then it is solved using the proposed method. A numerical example illustrates the proposed method.

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