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Shehu Transform of Error Function (Probability Integral)

Sudhanshu Aggarwal, Gyanvendra Pratap Singh

Abstract— The solutions of many advanced engineering problems like Fick's second law, heat and mass transfer problems, vibrating beams problems contains error and complementary error function. When we use any integral transform to solve these types of problems, it is very necessary to know the integral transform of error function. In this article, we find the Shehu transform of error and complementary error functions. To demonstrate the usefulness of Shehu transform of error function, some numerical applications are considered in application section for solving improper integrals which contain error function. It is pointed out that Shehu transform give the exact solution of improper integral which contains error function without any tedious calculation work.

IndexTerms— Complementary error function, Error function, Improper integral, Shehu transform.

AMS Subject Classification: 44A05, 44A20, 44A35.

I. INTRODUCTION

Integral transforms are highly efficient for solving many advance problems of science and engineering such as radioactive decay problems, heat conduction problems, problem of motion of a particle under gravity, vibration problems of beam, electric circuit problems and population growth problems. Many researchers applied different integral transforms (Laplace transform [1-2], Fourier transform [2], Kamal transform [3-10, 48-49], Aboodh transform [11-16, 50-52], Mahgoub transform [17-25, 45-47], Mohand transform [26-29, 36, 53-56], Elzaki transform [37-40, 57-59], Shehu transform [41-43, 60] and Sumudu transform [44, 61-62]) and solved differential equations, delay differential equations, partial differential equations, integral equations, integro-differential equations and partial integro-differential equations. Sudhanshu et al. [30-35] discussed the comparative study of Mohand and other transforms (Laplace transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform and Mahgoub transform).

Error function occurs frequently in probability, physics, thermodynamics, statistics, mathematics and many engineering problems like heat conduction problems, vibrating beams problems etc. The error function is also known as the probability integral. The error function is a special function because it cannot be evaluated by usual methods of integration. Mathematically error and complimentary error functions are defined by [63-68]

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \qquad (1)$$

and

$$erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt \qquad (2)$$

The Shehu transform of the function $F(t)$ for all $t \ge 0$ is defined as [60]:

$$S\{F(t)\} = \int_0^\infty F(t)e^{-\frac{vt}{u}}dt = H(v,u), v>0, u>0, \qquad (3)$$

where operator S is called the Shehu transform operator.

The main purpose of the present article is to determine Shehu transform of error function and explain the importance of Shehu transform of error function by giving some numerical applications in application section of this paper.

II. SOME USEFUL PROPERTIES OF SHEHU TRANSFORM

2.1 Linearity property [41-43]:

If Shehu transform of functions $F_1(t)$ and $F_2(t)$ are $H_1(v,u)$ and $H_2(v,u)$ respectively then Shehu transform of $[aF_1(t)+bF_2(t)]$ is given by $[a\,H_1(v,u)+bH_2(v,u)]$, where a,b are arbitrary constants.

Proof: By the definition of Shehu transform, we have

$$S\{F(t)\} = \int_0^\infty F(t)e^{-\frac{vt}{u}}dt$$

$$\Rightarrow S\{aF_1(t)+bF_2(t)\} = \int_0^\infty [aF_1(t)+bF_2(t)]e^{-\frac{vt}{u}}dt$$

$$\Rightarrow S\{aF_1(t)+bF_2(t)\}$$

$$= a\int_0^\infty F_1(t)e^{-\frac{vt}{u}}dt + b\int_0^\infty F_2(t)e^{-\frac{vt}{u}}dt$$

$$\Rightarrow S\{aF_1(t)+bF_2(t)\} = aS\{F_1(t)\}+bS\{F_2(t)\}$$

$$\Rightarrow S\{aF_1(t)+bF_2(t)\} = a\,H_1(v,u)+bH_2(v,u),$$

where a,b are arbitrary constants.

2.2 Change of scale property:

If Shehu transform of function $F(t)$ is $H(v,u)$ then Shehu transform of function $F(at)$ is given by $\frac{1}{a}H\left(\frac{v}{a},u\right)$.

Proof: By the definition of Shehu transform, we have

$$S\{F(at)\} = \int_0^\infty F(at)e^{-\frac{vt}{u}}dt \qquad (4)$$

Put $at = p \Rightarrow adt = dp$ in equation(4), we have

$$S\{F(at)\} = \frac{1}{a}\int_0^\infty F(p)e^{\frac{-vp}{ua}}dp$$

$$\Rightarrow S\{F(at)\} = \frac{1}{a}\left[\int_0^\infty F(p)e^{\frac{-(v/a)p}{u}}dp\right]$$

$$\Rightarrow S\{F(at)\} = \frac{1}{a}H\left(\frac{v}{a},u\right)$$

2.3 Shifting property:

If Shehu transform of function $F(t)$ is $H(v,u)$ then Shehu transform of function $e^{at}F(t)$is given by $H(v-au,u)$.

Proof: By the definition of Shehu transform, we have

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Sudhanshu Aggarwal, *Department of Mathematics, National P.G. College Barhalganj, Gorakhpur-273402, U.P., India.*
Gyanvendra Pratap Singh, *Department of Mathematics and Statistics, D.D.U. Gorakhpur University, Gorakhpur-273009, U.P., India.*

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$$S\{e^{at}F(t)\} = \int_0^\infty e^{at}F(t)e^{-\frac{vt}{u}}dt$$
$$\Rightarrow S\{e^{at}F(t)\} = \int_0^\infty F(t)e^{-\left(\frac{v}{u}-a\right)t}dt$$
$$= \int_0^\infty F(t)e^{-\left(\frac{v-au}{u}\right)t}dt = H(v-au,u)$$

2.4 Shehu transform of the derivatives of the function $F(t)$ [41-43]:

If $S\{F(t)\} = H(v,u)$ then

a) $S\{F'(t)\} = \frac{v}{u}H(v,u) - F(0)$

b) $S\{F''(t)\} = \frac{v^2}{u^2}H(v,u) - \frac{v}{u}F(0) - F'(0)$

c) $S\{F^{(n)}(t)\} = \frac{v^n}{u^n}H(v,u) - \sum_{k=0}^{n-1}\left(\frac{v}{u}\right)^{n-(k+1)}F^{(k)}(0)$

2.5 Shehu transform of integral of a function $F(t)$:

If $S\{F(t)\} = H(v,u)$ then $S\left\{\int_0^t F(t)dt\right\} = \frac{u}{v}H(v,u)$.

Proof: Let $G(t) = \int_0^t F(t)dt$. Then $G'(t) = F(t)$ and $G(0) = 0$.

Now by the property of Shehu transform of the derivative of function, we have

$$S\{G'(t)\} = \frac{v}{u}S\{G(t)\} - G(0) = \frac{v}{u}S\{G(t)\}$$
$$\Rightarrow S\{G(t)\} = \frac{u}{v}S\{G'(t)\} = \frac{u}{v}\,S\{F(t)\}$$
$$\Rightarrow S\{G(t)\} = \frac{u}{v}H(v,u)$$
$$\Rightarrow S\left\{\int_0^t F(t)dt\right\} = \frac{u}{v}H(v,u)$$

2.6 Convolution theorem for Shehu transforms [42-43]:

If Shehu transform of functions $F_1(t)$ and $F_2(t)$ are $H_1(v,u)$ and $H_2(v,u)$ respectively then Shehu transform of their convolution $F_1(t) * F_2(t)$ is given by

$$S\{F_1(t) * F_2(t)\} = S\{F_1(t)\}S\{F_2(t)\}$$
$$\Rightarrow S\{F_1(t) * F_2(t)\} = H_1(v,u)H_2(v,u),$$

where $F_1(t) * F_2(t)$ is defined by

$$F_1(t) * F_2(t) = \int_0^t F_1(t-x)\,F_2(x)dx$$
$$= \int_0^t F_1(x)\,F_2(t-x)dx.$$

Proof: By the definition of Shehu transform, we have

$$S\{F_1(t) * F_2(t)\} = \int_0^\infty e^{-\frac{vt}{u}}[F_1(t) * F_2(t)]dt$$
$$\Rightarrow S\{F_1(t) * F_2(t)\} = \int_0^\infty e^{-\frac{vt}{u}}\left[\int_0^t F_1(t-x)\,F_2(x)dx\right]dt$$

By changing the order of integration, we have

$$S\{F_1(t) * F_2(t)\} = \int_0^\infty F_2(x)\left[\int_x^\infty e^{-\frac{vt}{u}}F_1(t-x)\,dt\right]dx$$

Put $t - x = p$ so that $dt = dp$ in above equation, we have

$$S\{F_1(t) * F_2(t)\} = \int_0^\infty F_2(x)\left[\int_0^\infty e^{-\frac{v(p+x)}{u}}F_1(p)\,dp\right]dx$$
$$\Rightarrow S\{F_1(t) * F_2(t)\} = \int_0^\infty F_2(x)e^{-\frac{xv}{u}}\left[\int_0^\infty e^{-\frac{pv}{u}}F_1(p)\,dp\right]dx$$
$$\Rightarrow S\{F_1(t) * F_2(t)\} = H_1(v,u)H_2(v,u).$$

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III. SHEHU TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS [41-43]

Table: 1

S.N.	$F(t)$	$S\{F(t)\} = H(v,u)$
1.	1	$\frac{u}{v}$
2.	t	$\left(\frac{u}{v}\right)^2$
3.	t^2	$2!\left(\frac{u}{v}\right)^3$
4.	$t^n, n \in N$	$n!\left(\frac{u}{v}\right)^{n+1}$
5.	$t^n, n > -1$	$\Gamma(n+1)\left(\frac{u}{v}\right)^{n+1}$
6.	e^{at}	$\frac{u}{v-au}$
7.	$sinat$	$\frac{au^2}{(v^2+a^2u^2)}$
8.	$cosat$	$\frac{uv}{(v^2+a^2u^2)}$
9.	$sinhat$	$\frac{au^2}{(v^2-a^2u^2)}$
10.	$coshat$	$\frac{uv}{(v^2-a^2u^2)}$
11	$J_0(at)$	$\frac{u}{\sqrt{(v^2+a^2u^2)}}$

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IV. SOME IMPORTANT PROPERTIES OF ERROR AND COMPLEMENTARY ERROR FUNCTIONS

4.1 The sum of error and complementary error functions is unity:

$erf(x) + erfc(x) = 1$

Proof: we have $\int_0^\infty e^{-t^2}dt = \frac{\sqrt{\pi}}{2}$

$$\Rightarrow \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-t^2}dt = 1$$
$$\Rightarrow \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt + \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt = 1$$
$$\Rightarrow erf(x) + erfc(x) = 1$$

4.2 Error function is an odd function:

$erf(-x) = -erf(x)$

4.3 The value of error function at $x = 0$ is 0:

$erf(0) = 0.$

4.4 The value of complementary error function at $x = 0$ is 1:

$erfc(0) = 1.$

4.5 The domain of error and complementary error functions is $(-\infty, \infty)$.

4.6 $erf(x) \to 1 as\ x \to \infty$.

4.7 $erfc(x) \to 0 as\ x \to \infty$.

4.8 The value of error functions $erf(x)$ for different values of x [64]:

Table: 2

S.N.	x	$erf(x)$

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1.	0.00	0.00000
2.	0.02	0.02256
3.	0.04	0.04511
4.	0.06	0.06762
5.	0.08	0.09008
6.	0.10	0.11246
7.	0.12	0.13476
8.	0.14	0.15695
9.	0.16	0.17901
10.	0.18	0.20094
11.	0.20	0.22270

V. SHEHU TRANSFORM OF ERROR FUNCTION

By equation (1), we have

$$erf(\sqrt{t}) = \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-x^2}dx$$

$$= \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}}\left[1-\frac{x^2}{1!}+\frac{x^4}{2!}-\frac{x^6}{3!}+\cdots\ldots\right]dx$$

$$= \frac{2}{\sqrt{\pi}}\left[x-\frac{x^3}{3.1!}+\frac{x^5}{5.2!}-\frac{x^7}{7.3!}+\cdots\ldots\right]_0^{\sqrt{t}}$$

$$= \frac{2}{\sqrt{\pi}}\left[t^{1/2}-\frac{t^{3/2}}{3.1!}+\frac{t^{5/2}}{5.2!}-\frac{t^{7/2}}{7.3!}+..\right] \quad (5)$$

Applying Shehu transform both sides on equation (5), we get

$$S\{erf(\sqrt{t})\} = \frac{2}{\sqrt{\pi}}S\left\{\left[t^{1/2}-\frac{t^{3/2}}{3.1!}+\frac{t^{5/2}}{5.2!}-\frac{t^{7/2}}{7.3!}+..\right]\right\} \quad (6)$$

Applying the linearity property of Shehu transform on equation (6), we get

$$S\{erf(\sqrt{t})\} = \frac{2}{\sqrt{\pi}}\left[\Gamma\left(\frac{3}{2}\right)\left(\frac{u}{v}\right)^{\frac{3}{2}}-\frac{\Gamma\left(\frac{5}{2}\right)}{3.1!}\left(\frac{u}{v}\right)^{\frac{5}{2}}+\frac{\Gamma\left(\frac{7}{2}\right)}{5.2!}\left(\frac{u}{v}\right)^{\frac{7}{2}}-\frac{\Gamma\left(\frac{9}{2}\right)}{7.3!}\left(\frac{u}{v}\right)^{\frac{9}{2}}+\cdots\right]$$

$$= \frac{2}{\sqrt{\pi}}\Gamma(3/2)\left(\frac{u}{v}\right)^{3/2}\left[1-\frac{1}{2}\left(\frac{u}{v}\right)+\frac{1.3}{2.4}\left(\frac{u}{v}\right)^2-\frac{1.3.5}{2.4.6}\left(\frac{u}{v}\right)^3+\cdots\ldots\ldots\right]$$

$$= \left(\frac{u}{v}\right)^{3/2}\left(1+\frac{u}{v}\right)^{-1/2} = \frac{u^{3/2}}{v\sqrt{(u+v)}} \quad (7)$$

VI. SHEHU TRANSFORM OF COMPLEMENTARY ERROR FUNCTION

We have, $erf(\sqrt{t})+erfc(\sqrt{t}) = 1$

$$\Rightarrow erfc(\sqrt{t}) = 1-erf(\sqrt{t}) \quad (8)$$

Applying Shehu transform both sides on equation (8), we have

$$S\{erfc(\sqrt{t})\} = S\{1-erf(\sqrt{t})\} \quad (9)$$

Applying the linearity property of Shehu transform on equation (9), we get

$$S\{erfc(\sqrt{t})\} = S\{1\}-S\{erf(\sqrt{t})\}$$

$$\Rightarrow S\{erfc(\sqrt{t})\} = \frac{u}{v}-\frac{u^{3/2}}{v\sqrt{(u+v)}}$$

$$\Rightarrow S\{erfc(\sqrt{t})\} = \frac{u}{v}\left[\frac{\sqrt{(u+v)}-u^{1/2}}{\sqrt{(u+v)}}\right] \quad (10)$$

VII. APPLICATIONS

In this section, some applications are given in order to explain the advantage of Shehu transform of error function for evaluating the improper integral, which contain error function.

7.1 Evaluate the improper integral

$$I = \int_0^\infty e^{-t}\,erf(\sqrt{t})\,dt.$$

We have $S\{erf(\sqrt{t})\} = \int_0^\infty erf(\sqrt{t})\,e^{-\frac{vt}{u}}dt$

$$\Rightarrow S\{erf(\sqrt{t})\} = \frac{u^{3/2}}{v\sqrt{(u+v)}} \quad (11)$$

Taking $\left(\frac{v}{u}\right)\to 1$ in above equation, we have

$$I = \int_0^\infty e^{-t}\,erf(\sqrt{t})\,dt = \frac{1}{\sqrt{2}}$$

7.2 Evaluate the improper integral

$$I = \int_0^\infty e^{-(v-2)t}\,erf(\sqrt{t})\,dt.$$

We have $S\{erf(\sqrt{t})\} = \frac{u^{3/2}}{v\sqrt{(u+v)}}$

Now by shifting theorem of Shehu transform, we have

$$S\{e^{2t}\,erf(\sqrt{t})\} = \left[\frac{u^{3/2}}{(v-2)\sqrt{(u+(v-2))}}\right]$$

$$\Rightarrow S\{e^{2t}\,erf(\sqrt{t})\} = \frac{u^{3/2}}{(v-2)\sqrt{(u+v-2)}} \quad (12)$$

By the definition of Shehu transform, we have

$$S\{e^{2t}\,erf(\sqrt{t})\} = \int_0^\infty e^{2t}\,erf(\sqrt{t})\,e^{-\frac{vt}{u}}dt$$

$$\Rightarrow S\{e^{2t}\,erf(\sqrt{t})\} = \int_0^\infty e^{-(v/u-2)t}\,erf(\sqrt{t})\,dt \quad (13)$$

Now by equations (12) and (13), we get

$$\int_0^\infty e^{-(v/u-2)t}\,erf(\sqrt{t})\,dt = \frac{u^{3/2}}{(v-2)\sqrt{(u+v-2)}}$$

Taking $u\to 1$ in above equation, we have

$$\Rightarrow I = \int_0^\infty e^{-(v-2)t}\,erf(\sqrt{t})\,dt = \frac{1}{(v-2)\sqrt{(v-1)}}.$$

7.3 Evaluate the improper integral

$$I = \int_0^\infty e^{-t}\left\{\int_0^t erf(\sqrt{x})dx\right\}dt.$$

We have $S\{erf(\sqrt{t})\} = \frac{u^{3/2}}{v\sqrt{(u+v)}}$

Now by the property of Shehu transform of integral of a function, we have

$$S\left\{\int_0^t erf(\sqrt{x})dx\right\} = \frac{u}{v}\left[\frac{u^{3/2}}{v\sqrt{(u+v)}}\right]$$

$$\Rightarrow S\left\{\int_0^t erf(\sqrt{x})dx\right\} = \frac{u^{5/2}}{v^2\sqrt{(u+v)}} \quad (14)$$

By the definition of Shehu transform, we have

$$S\left\{\int_0^t erf(\sqrt{x})dx\right\} = \int_0^\infty e^{-\frac{vt}{u}}\left\{\int_0^t erf(\sqrt{u})du\right\}dt \quad (15)$$

Now by equations (14) and (15), we get

$$\int_0^\infty e^{-\frac{vt}{u}}\left\{\int_0^t erf(\sqrt{x})dx\right\}dt = \frac{u^{5/2}}{v^2\sqrt{(u+v)}}$$

Taking $\left(\frac{v}{u}\right)\to 1$ in above equation, we have

$$I = \int_0^\infty e^{-t}\left\{\int_0^t erf(\sqrt{x})dx\right\}dt = \frac{1}{\sqrt{2}}.$$

7.4 Evaluate the improper integral

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$$I = \int_0^\infty e^{-2t}\left[\frac{d}{dt} erf\left(2\sqrt{t}\right)\right] dt.$$

We have $S\{erf(\sqrt{t})\} = \frac{u^{3/2}}{v\sqrt{(u+v)}}$

Now by change of scale property of Shehu transform, we have

$$S\{erf(2\sqrt{t})\} = \frac{1}{4}\left[\frac{u^{3/2}}{(v/4)\sqrt{(u+v/4)}}\right]$$

$$\Rightarrow S\{erf(2\sqrt{t})\} = \frac{2u^{3/2}}{v\sqrt{(4u+v)}}$$

Now using the property of Shehu transform of derivative of a function, we have

$$S\left\{\frac{d}{dt} erf(2\sqrt{t})\right\} = \frac{v}{u}.\left[\frac{2u^{3/2}}{v\sqrt{(4u+v)}}\right] - 0$$

$$\Rightarrow S\left\{\frac{d}{dt} erf(2\sqrt{t})\right\} = \frac{2u^{1/2}}{\sqrt{(4u+v)}} \quad (16)$$

By the definition of Shehu transform, we have

$$S\left\{\frac{d}{dt} erf(2\sqrt{t})\right\} = \int_0^\infty e^{-\frac{vt}{u}}\left\{\frac{d}{dt} erf(2\sqrt{t})\right\} dt \quad (17)$$

Now by equations (16) and (17), we get

$$\int_0^\infty e^{-\frac{vt}{u}}\left\{\frac{d}{dt} erf(2\sqrt{t})\right\} dt = \frac{2u^{1/2}}{\sqrt{(4u+v)}}$$

Taking $\left(\frac{v}{u}\right) \to 2$ in above equation, we have

$$\int_0^\infty e^{-2t}\left\{\frac{d}{dt} erf(2\sqrt{t})\right\} dt = \frac{2}{\sqrt{6}}$$

$$\Rightarrow I = \int_0^\infty e^{-2t}\left\{\frac{d}{dt} erf(2\sqrt{t})\right\} dt = \frac{2}{\sqrt{6}}$$

$$\Rightarrow I = \int_0^\infty e^{-2t}\left\{\frac{d}{dt} erf(2\sqrt{t})\right\} dt = \sqrt{\frac{2}{3}}.$$

7.5 Evaluate the improper integral

$$I = \int_0^\infty e^{-5t}\left[erf\sqrt{t} * erf\sqrt{t}\right] dt.$$

By convolution theorem of Shehu transform, we have

$$S\{erf\sqrt{t} * erf\sqrt{t}\} = S\{erf(\sqrt{t})\}S\{erf(\sqrt{t})\}$$

$$= \left[\frac{u^{3/2}}{v\sqrt{(u+v)}}\right]\left[\frac{u^{3/2}}{v\sqrt{(u+v)}}\right] = \frac{u^3}{v^2(u+v)} \quad (18)$$

Now by the definition of Shehu transform, we have

$$S\{erf\sqrt{t} * erf\sqrt{t}\} = \int_0^\infty e^{-\frac{vt}{u}}\{erf\sqrt{t} * erf\sqrt{t}\} dt \quad (19)$$

Now by equations (18) and (19), we get

$$\int_0^\infty e^{-\frac{vt}{u}}\{erf\sqrt{t} * erf\sqrt{t}\} dt = \frac{u^3}{v^2(u+v)} \quad (20)$$

Taking $\left(\frac{v}{u}\right) \to 5$ in above equation, we have

$$\int_0^\infty e^{-5t}\{erf\sqrt{t} * erf\sqrt{t}\} dt = \frac{1}{150}$$

$$\Rightarrow I = \int_0^\infty e^{-5t}\{erf\sqrt{t} * erf\sqrt{t}\} dt = \frac{1}{150}.$$

VIII. CONCLUSION

In this article, we have successfully discussed the Shehu transform of error function. The given numerical applications in application section show the advantage of Shehu transform of error function for evaluating the improper integral, which contain error function. Results of numerical applications show Shehu transform give the exact solution without any tedious calculation work. In future, Shehu transform can be used in solving vibrating beam problems, heat and mass transfer problems.

CONFLICT OF INTEREST

The authors confirm that this article contents have no conflict of interest.

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AUTHORS PROFILE

Sudhanshu Aggarwal received his M.Sc. degree from M.S. College, Saharanpur in 2007. He has also qualified CSIR NET examination (June-2010, June-2012, June-2013, June-2014 & June-2015) in Mathematical Sciences. He is working as an Assistant Professor in National P.G. College Barhalganj Gorakhpur. He is equipped with an extraordinary caliber and appreciable academic potency. His fields of interest include Integral Transform Methods, Differential and Partial Differential Equations, Integral Equations, Number Theory, Vibration of Plates and Theory of Elasticity. He has published many research papers in national and international journals.

Dr. Gyanvendra Pratap Singh is working as an Assistant Professor in Department of Mathematics and Statistics, D.D.U. Gorakhpur University, Gorakhpur. He has completed her Ph.D degree from D.D.U. Gorakhpur University. He has published many research papers in reputed journals. His area of interest is Integral Transform and Finsler Geometry.

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