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η−dual of Generalized Difference Sequence Spaces

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Abstract. The notion of $\alpha-$ Köthe Toeplitz dual was generalized by Tripathy and Chandra [11] to introduce $\eta-$dual. In this paper we give $\eta-$dual of sequence spaces $\Delta^m_{v,s}(l_\infty)$, $\Delta^m_{v,s}(c)$ and $\Delta^m_{v,s}(c_0)$.

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1. INTRODUCTION

Let l_∞, c and c_0 be the linear spaces of bounded convergent and null sequences $x=(x_k)$ with complex term respectively norm by

$$\| x \|_\infty = \sup_k | x_k |$$

where $k \in N = \{1,2,3,\ldots\}$ the set of positive integer.

In 1981, Kizmaz [8] introduce the concept of difference sequence and have defined $\Delta-$bounded, $\Delta-$convergent and $\Delta-$null sequence spaces. Using the concept of difference sequence Cólak [3] has defined Δ^m- bounded Δ^m-convergent and Δ^m-null sequence spaces. Further this notion was generalized by Et. and Esi [5] and have defined Δ^m_v- bounded, Δ^m_v- convergent and Δ^m_v- null sequence spaces, where $v=(v_k)$ be any fixed sequence of non-zero complex number's. Later on Bektas and Cólak [2] have defined Δ^m_r- bounded, Δ^m_r- convergent and Δ^m_r- null sequence spaces.

Recently Ansari and Chaudhary [1] have defined the following sequence spaces.

Let $v=(v_k)$ be any fixed sequence of non-zero complex number, then

$$\Delta^m_{v,s}(l_\infty) = \{x=(x_k) : (k^s \Delta^m_v x_k) \in l_\infty\}$$

$$\Delta^m_{v,s}(c) = \{x=(x_k) : (k^s \Delta^m_v x_k) \in c\}$$

$$\Delta^m_{v,s}(c_0) = \{x=(x_k) : (k^s \Delta^m_v x_k) \in c_0\}$$

where $m \in N$, $s \in R$,

$$\Delta^m_{v,s}(x) = (k^s \Delta^m_v x_k) = k^s (\Delta^{m-1}_v x_k - \Delta^{m-1}_v x_{k+1})$$

and $\Delta^m_v x_k = \sum_{j=1}^{m} (-1)^j (j^m) v_{k+j} x_{k+j}$.

These are Banach spaces with norm

$$\| x \|_{v,s} = \sum_{c=1}^{m} |v_i x_i| + \sup_k | k^r \Delta^m_v x_k |.$$

It is trivial that $c_0(\Delta^m_s) \subset c_0(\Delta^{m+1}_s)$, $c(\Delta^m_s) \subset c(\Delta^{m+1}_s)$, $l_\infty(\Delta^m_s) \subset l_\infty(\Delta^{m+1}_s)$ and $c_0(\Delta^m_s) \subset c(\Delta^m_s) \subset l_\infty(\Delta^m_s)$.

Lemma 1.1. [1] $\sup_k k^s |\Delta^m_v x_k| < \infty$ iff

(i) $\sup_k k^{s-1} |\Delta^{m-1}_v x_k| < \infty$

(ii) $\sup_k k^s |\Delta^{m-1}_v x_k - k(k+1)^{-1} \Delta^{m-1}_v x_{k+1}| < \infty$

Corollary 1.2. [1] $x \in \Delta^m_{v,s}(l_\infty)$ implies $\sup_k k^{s-m} |v_k x_k| < \infty$.

2. MAIN RESULTS

Definition 2.1. [11] Let E be a sequence space, then the $\eta-$dual of E is defined as

$$E^\eta = \{a=(a_k) : \sum |a_k x_k|^r < \infty, r \geq 1\}.$$

Definition 2.2. [11] Let E be a sequence space. Then E is called a perfect space iff $E = E^{\eta\eta}$.

Lemma 2.3. [11]

(i) E^η is a linear subspace of w for $E \subset w$

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(ii) $E \subset F$ implies $E^{\eta} \supset F^{\eta}$ for every $E, F \subset w$

(iii) $(E^{\eta})^{\eta} = E^{\eta\eta} \supset E$ for every $E \subset w$

(iv) $(\bigcup_j E_j)^{\eta} = \bigcap_j E_j^{\eta}$ for every family $\{E_j\}$ with $E_j \subset w$ for all $j \in N$.

Theorem 2.4. Let *m* be a positive integer and $s \in R$, we put

$$M_{\eta}(v,s) = \{a = (a_k) : \sum_{k=1}^{\infty} (k^{m-s})^r \mid a_k v_k^{-1} \mid^r < \infty\}.$$

Then,

$$[\Delta_{v,s}^m(l_\infty)]^{\eta} = [\Delta_{v,s}^m(c)]^{\eta} = [\Delta_{v,s}^m(c_0)]^{\eta} = M_{\eta}(v,s). \quad (2.1)$$

Proof. First we assume that $a \in M_{\eta}(v,s)$. Then

$$| a_k x_k |^r = | k^{m-s} a_k v_k^{-1} k^{s-m} x_k v_k |^r = (k^{m-s})^r \mid a_k v_k^{-1} \mid^r \mid k^{s-m} x_k v_k \mid^r$$
or

$$\sum_{k=1}^{\infty} |a_k x_k|^r = \sum_{k=1}^{\infty} (k^{m-s})^r \mid a_k v_k^{-1} \mid^r \mid k^{s-m} x_k v_k \mid^r < \infty$$ for each $x \in \Delta_{v,s}^m(l_\infty)$, by corollary 1.2. Thus, we have to shown

$$M_{\eta}(v,s) \subset [\Delta_{v,s}^m(l_\infty)]^{\eta}, \quad (2.2)$$

conversely, let $a \notin M_{\eta}(v,s)$, then for some k, we have

$$\sum_{k=1}^{\infty} (k^{m-s})^r \mid a_k v_k^{-1} \mid^r = \infty.$$

So, there is a strictly increasing sequence (n_i) of positive integer n_i, such that

$$\sum_{k=n_i+1}^{n_i+1} (k^{m-s})^r \mid a_k v_k^{-1} \mid^r > i^r.$$

We defined as a sequence $x = (x_k)$ by

$$x_k = \begin{cases} 0 & (1 \le k \le n_k) \\ \dfrac{v_k^{-1} k^{m-s}}{i^r} & (n_i + 1 < k \le n_{i+1} : i = 1, 2, \ldots). \end{cases}$$

Then, we see that

$$k^s \mid \Delta^m v_k x_k \mid = \frac{m!}{i^r} (n_i + 1 < k \le n_{i+1}, i = 1, 2, \ldots).$$

Hence,

$$x \in \Delta_{v,s}^m(c_0) \quad \text{and} \quad \sum_{k=1}^{\infty} |a_k x_k| > \sum 1 = \infty.$$

Thus, $a \notin [\Delta_{v,s}^m(l_\infty)]^{\eta}$, and hence, we have shown

$$[\Delta_{v,s}^m(c_0)]^{\eta} \subset M_{\eta}(v,s). \quad (2.3)$$

Since

$$\Delta_{v,s}^m(c_0) \subset \Delta_{v,s}^m(c) \subset \Delta_{v,s}^m(l_\infty)$$

implies

$$[\Delta_{v,s}^m(l_\infty)]^{\eta} \subset [\Delta_{v,s}^m(c)]^{\eta} \subset [\Delta_{v,s}^m(c_0)]^{\eta}$$

(2.1) follows from (2.2) and (2.3).

Theorem 2.5. Let *m* be a positive integer and $s \in R$, we put

$M_{\eta\eta} = \{a = (a_k) : \sup_k (k^{s-m})^r \mid a_k v_k \mid^r < \infty\}$. Then

$$[\Delta_{v,s}^m(l_\infty)]^{\eta\eta} = [\Delta_{v,s}^m(c)]^{\eta\eta} = [\Delta_{v,s}^m(c_0)]^{\eta\eta} = M_{\eta\eta}(v,s). \quad (2.4)$$

Proof. First we assume that $a \in M_{\eta\eta}(v,s)$. Then

$$\begin{aligned} | a_k x_k |^r &= | k^{s-m} a_k v_k k^{m-s} x_k v_k^{-1} |^r \\ &= (k^{s-m})^r \mid a_k v_k \mid^r (k^{m-s})^r \mid x_k v_k^{-1} \mid^r \end{aligned}$$ or,

$$\sum_{k=1}^{\infty} |a_k x_k|^r < \sup_k (k^{s-m})^r \mid a_k v_k \mid^r \sum_{k=1}^{\infty} (k^{m-s})^r \mid x_k v_k^{-1} \mid^r < \infty$$ for each $x \in [\Delta_{v,s}^m(c_0)]^{\alpha} = M_{\eta}(v,s)$ by using (2.1). Thus, we have shown

$$M_{\eta\eta}(v,s) \subset [\Delta_{v,s}^m(c_0)]^{\eta\eta}. \quad (2.5)$$

Conversely, let $a \notin M_{\eta\eta}(v,s)$. Then, we have

$$\sup_k (k^{s-m})^r \mid a_k v_k \mid^r = \infty.$$

Hence, there is strictly increasing sequence $(k(i))$ of positive integer $k(i)$ such that

$$\{[k(i)]^{s-m}\}^r \mid a_{k(i)} v_{k(i)} \mid^r > i^{mr}, \qquad r > 1.$$

Then, we see that

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$$\sum_{k=1}^{\infty}(k^{m-s})^r \mid x_k v_k^{-1}\mid^r \quad = \sum_{i=1}^{\infty}\{[k(i)]^{m-s}\}^r \mid a_{k(i)} v_{k(i)}\mid^r$$
$$\leq \sum_{i=1}^{\infty} i^{-mr} < \infty.$$

Hence, $x \in [\Delta_{v,s}^m (l_\infty)]^\eta$ and

$$\sum_{k=1}^{\infty}|a_k x_k|^r = \sum_{i=1}^{\infty} 1 = \infty .$$

Thus $a \notin \Delta_{v,s}^m [\Delta_{v,s}^m (l_\infty)]^{\eta\eta}$ and hence we have to shown

$$[\Delta_{v,r}^m (l_\infty)]^{\eta\eta} \subset M_{\eta\eta}(v,s). \tag{2.6}$$

Since,

$$[\Delta_{v,s}^m (l_\infty)]^\eta \subset [\Delta_{v,s}^m (c)]^\eta \subset [\Delta_{v,s}^m (c_0)]^\eta$$

implies

$$[\Delta_{v,s}^m (c_0)]^{\eta\eta} \subset [\Delta_{v,s}^m (c)]^{\eta\eta} \subset [\Delta_{v,s}^m (l_\infty)]^{\eta\eta},$$

(2.4) follows from (2.5) and (2.6).

By definition 2.2., we also have

Corollary 2.6. The sequence spaces $\Delta_{v,s}^m (l_\infty)$, $\Delta_{v,s}^m (c)$ and $\Delta_{v,s}^m (c_0)$ are not perfect.

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