# Dynamic State Performance Analysis of Three Phase Transfer Field Machine with Rotor (CAGE) Windings 

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#### Abstract

The new configuration of the conventional (existing) three phase transfer field reluctance machine is intended to minimize the excessive leakage reactance associated with the quadrature axis reactance of the machine, due to salient nature of its rotor poles structure. This is achieved by optimizing the rotor design, through the introduction of cage (rotor) windings at the periphery of the shaft of the machine sections (M/C A and M/C B). The idea is that when windings are wounded on the rotor sections of the machine, resistance is added to the rotor circuit, the rotor power factor is improved, which in turn results in improved output torque. This of course increases the rotor impedance and therefore decreases the value of rotor circuit. Consequently, the effect of improved power factor predominates and the starting torque is increased. To cushion the effect such reduction in induced rotor current, the auxiliary and rotor (cage) windings of both machines sections are connected in parallel but transposed between the two machine halves and short circuited.


Index Terms- Auxiliary windings, Cage (rotor) windings, induced rotor current, main windings, resultant impedance.

## I. Introduction

The scale or yard-stick upon which every electric motor is measured is principally the nature of its output characteristics. Hence, the consistence research on the improvement of motor output characteristics, which includes output power, electromagnetic torque, power factor, efficiency etc becomes pertinent. The output of all plain transfer field effect machines would be less than that of a conventional induction machine of comparable size and rating. This is the attribute of their low direct axis reactance to quadrature axis reactance ratio, accompanied with excessive leakage reactance (Agu L.A, 1984). An analysis on improvement in this disorderliness is the subject matter of this novel.

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## II. The Machine Description

The structural arrangement of the machine under study is shown in figure 1. Unlike the existing three phase transfer field cage -less machine counterpart (Agu L.A, Anih L.U 2002), the three-phase transfer field machine with cage windings comprise two identical poly-phase reluctance machine with moving conductors (rotor windings), whose salient poles rotor are mechanically coupled together, such that their $\mathbf{d}$ and $\mathbf{q}$ axes are in space quadrature. As depicted in figure 1, the stator (primary) windings, are integrally wound. Each machine element has three sets of windings. Both sets of windings are identical. The main and auxiliary windings are housed in the stators slots. The main windings of the machine carry the excitation current, while the auxiliary and the rotor windings, carry the circulating current. The ( $2 \mathrm{~s}-1$ ) $\omega_{0}$ low frequency current is confined in the auxiliary and rotor windings without interfering with the supply. The main windings of the machine sets are connected in series while the auxiliary windings, though also in the stator are transposed between the two machine stacks. They are wound for the same pole number and both are star connected. The third set of windings known as the rotor (cage) windings are wounded at the periphery of the rotor shaft connecting the two machine sets. Just as in the auxiliary windings, the rotor (cage) windings are also transposed between the two machine stacks and then connected in parallel with the auxiliary windings (see figure 2).


Figure 1: Connection diagram for three phase transfer field reluctance machine with rotor (cage) winding


Figure 2 Per phase schematic diagram of 3-phase transfer field motor with rotor (cage) windings.

## III. Dynamic Model Of 3-Phase Transfer Field Machine With Cage (Rotor) Windings

For us to derive the dynamic equations of the circuit model of 3-Phase transfer Field Machine with cage (rotor) windings, it is paramount to take a look at the variation of inductances with rotor position since the rotor has salient poles. In general, the peameance along the $\mathbf{d}$ and $\mathbf{q}$ axes is not the same.
Since the rotor is of salient poles, its mmfs are always directed along the $\mathbf{d}$ and $\mathbf{q}$ axes. Also, the direction of the resultant mmf of the stator windings relative to these two axes will vary with the power factor. A common approach to handling the magnetic effect of the stator's resultant mmf is to resolve it along the $\mathbf{d}$ and $\mathbf{q}$ axes, where it could be dealt with systematically. Let us consider the magnetic effect of current flowing in phase a of the stator. The resolved components of the a-phase $\operatorname{mmf~}_{\mathrm{a}}$, will produce the flux components;
$\phi_{\mathrm{d}}=\mathrm{P}_{\mathrm{d}} \mathrm{F}_{\mathrm{a}} \sin \theta_{\mathrm{r}}$ and $\phi_{\mathrm{q}}=\mathrm{P}_{\mathrm{q}} \mathrm{F}_{\mathrm{a}} \cos \theta_{\mathrm{r}}$ along the d and q axes respectively.

## Where $\mathrm{P}=$ peameance

The linkage of these resolved flux components with the a-phase windings is;
$\lambda_{\mathrm{aa}}=\mathrm{N}_{\mathrm{s}}\left(\phi_{\mathrm{d}} \operatorname{Sin} \theta_{\mathrm{r}}+\phi_{\mathrm{q}} \cos \theta_{\mathrm{r}}\right) \mathrm{Wb}$ turn.
$=N_{\mathrm{s}} \mathrm{F}_{\mathrm{a}}\left(\mathrm{P}_{\mathrm{d}} \sin ^{2} \theta_{\mathrm{r}}+\mathrm{P}_{\mathrm{q}} \cos ^{2} \theta_{\mathrm{r}}\right)$
$=\mathrm{N}_{\mathrm{s}} \mathrm{F}_{\mathrm{a}}\left(\frac{p_{d}+p_{q}}{2}-\frac{p_{d}-p_{q}}{2} \cos 2 \theta_{\mathrm{r}}\right)$

Similarly, the linkage of the flux component, $\phi_{\mathrm{d}}$ and $\phi_{\mathrm{q}}$ by the $\mathbf{b}$ - phase winding that is $\frac{2 \pi}{3}$ ahead may be written as: $\lambda_{b a}=N_{s} \mathrm{~F}_{\mathrm{a}}\left(\mathrm{P}_{\mathrm{d}} \sin \theta_{\mathrm{r}} \sin \left(\theta_{\mathrm{r}}-\frac{2 \pi}{3}\right)+\mathrm{p}_{\mathrm{q}} \cos \theta_{\mathrm{r}} \cos \left(\theta_{\mathrm{r}}-\frac{2 \pi}{3}\right)\right)$

$$
\begin{equation*}
=\mathrm{N}_{\mathrm{s}} \mathrm{~F}_{\mathrm{a}}\left(-\frac{p_{d}+p_{q}}{4}-\frac{p_{d}-p_{q}}{2} \cos 2\left(\theta_{\mathrm{r}}-\frac{\pi}{3}\right)\right) \tag{2}
\end{equation*}
$$

Based on the functional relationship of $\lambda_{\mathrm{aa}}$ with the rotor angle, $\theta_{\mathrm{r}}$, we can deduce that the self inductance of the stator a-phase winding, excluding the leakage has the form;

$$
\begin{aligned}
& \mathrm{L}_{\mathrm{aa}}=\mathrm{L}_{\mathrm{o}}-\mathrm{L}_{\mathrm{ms}} \cos 2 \theta_{\mathrm{r}} \mathrm{H} \\
& \text { Where; } \mathrm{L}_{\mathrm{o}}=\frac{L_{m d}+L_{m q}}{2} \text { and } \mathrm{L}_{\mathrm{ms}}=\frac{L_{m d}+L_{m q}}{2}
\end{aligned}
$$

Those of the $\mathbf{b}$ - and $\mathbf{c}$ - phases, $\mathrm{L}_{\mathrm{bb}}, \mathrm{L}_{\mathrm{cc}}$ are similar to that of $L_{a \mathrm{a}}$ but with $\theta_{\mathrm{r}}$ replaced by $\left(\theta_{\mathrm{r}}-\frac{2 \pi}{3}\right)$ and $\left(\theta_{\mathrm{r}}+\frac{2 \pi}{3}\right)$, respectively. Similarly, it can be deduced that the mutual inductance between the $\mathbf{a}$ and $\mathbf{b}$ phase of the stator is of the form,
$\mathrm{L}_{\mathrm{ab}}=\mathrm{L}_{\mathrm{ba}}=\frac{L_{o}}{2}-\mathrm{L}_{\mathrm{ms}} \cos 2\left(\theta_{\mathrm{r}}-\frac{2 \pi}{3}\right) \mathrm{H}$
Similarly, expression for $L_{b c}$ and $L_{a c}$ can be obtained by replacing $\theta_{\mathrm{r}}$ with $\left(\theta_{\mathrm{r}}-\frac{2 \pi}{3}\right)$ and $\left(\theta_{\mathrm{r}}+\frac{2 \pi}{3}\right)$ respectively.
Since a conventional (existing) transfer field effect machine is composed of two machine sets with two windings each (Agu L.A, 1978) if the parameter referring to the main winding is denoted with the subscript $\mathrm{A}, \mathrm{B}, \mathrm{C}$ (ie phase quantities) while that referring to the auxiliary winding will have subscript $\mathrm{a}, \mathrm{b}, \mathrm{c}$, the dynamic model can be derived as follows:

$V_{a b c}=r_{a b c} i_{a b c}+P \lambda_{a b c}$
$\mathrm{V}_{\mathrm{dqrABC}}=\mathrm{r}_{\mathrm{dqrABC}} \mathrm{i}_{\mathrm{dqrABC}}+\mathrm{P} \lambda_{\mathrm{dqrABC}}$
$V_{\text {dqrabc }}=\mathrm{r}_{\text {dqrabc }} \mathrm{i}_{\text {dqrabc }}+\mathrm{P} \lambda_{\text {dqrabc }}$
Where $\mathrm{P} \frac{d}{d t}, \lambda=$ flux
$R_{A B C}=\operatorname{diag}\left[\left(r_{A} r_{B} r_{C}\right)\right]$ and $r_{\text {abc }}=\operatorname{diag}\left[\left(r_{a} r_{b} r_{c}\right)\right]$
The flux linkages are expressed as;
$\lambda_{\mathrm{ABC}}=\mathrm{L}_{\mathrm{GG}} \mathrm{i}_{\mathrm{ABC}}+\mathrm{L}_{\mathrm{GH}} \mathrm{i}_{\mathrm{abc}}$
$\lambda_{\mathrm{abc}}=\mathrm{L}_{\mathrm{HG}} \mathrm{i}_{\mathrm{ABC}}+\mathrm{L}_{\mathrm{HH}} \mathrm{i}_{\mathrm{abc}}$
Where $\mathrm{L}_{\mathrm{GG}}, \mathrm{L}_{\mathrm{GH}}, \mathrm{L}_{\mathrm{HG}}$ and $\mathrm{L}_{\mathrm{HH}}$ are inductance matrices obtained from the inductance sub matrices of the two components machines as shown below.
Let $L_{11}$ be the self inductance of the main winding and $L_{22}$ be the self inductance of the auxiliary winding; then the mutual inductance between the main and the mutual inductance between the main and the auxiliary winding will be $\mathrm{L}_{12}$ or $\mathrm{L}_{21}$ as the case may be;

$$
\begin{aligned}
\text { Now; } \mathrm{L}_{11} & =\left[\begin{array}{lll}
L_{A A} & L_{A B} & L_{A C} \\
L_{B A} & L_{B B} & L_{B C} \\
L_{C A} & L_{C B} & L_{C C}
\end{array}\right] \\
\mathrm{L}_{12} & = \pm\left[\begin{array}{lll}
L_{A a} & L_{A b} & L_{A c} \\
L_{B a} & L_{B b} & L_{B c} \\
L_{C a} & L_{C h} & L_{C c}
\end{array}\right] \\
\mathrm{L}_{21} & = \pm\left[\begin{array}{lll}
L_{a A} & L_{a B} & L_{a c} \\
L_{b A} & L_{b B} & L_{b c} \\
L_{c A} & L_{c B} & L_{c C}
\end{array}\right] \\
\mathrm{L}_{12} & = \pm\left[\begin{array}{lll}
L_{a a} & L_{a b} & L_{a c} \\
L_{b a} & L_{b b} & L_{b c} \\
L_{c a} & L_{c h} & L_{c c}
\end{array}\right]
\end{aligned}
$$

So far the main and the auxiliary winding are identical,
$\mathrm{L}_{\mathrm{GG}}=\mathrm{L}_{11}($ Machine A$)+\mathrm{L}_{11}($ Machine B$)=L_{11}^{A}+L_{11}^{B}$
The individual inductance expressions are as follows;
$\mathrm{L}_{\mathrm{AA}}=\mathrm{L}_{\mathrm{a} 1}+\mathrm{L}_{\mathrm{a} 2} \cos 2 \theta_{\mathrm{r}}, \mathrm{L}_{\mathrm{AB}}=\mathrm{L}_{\mathrm{BA}}=-1 / 2 \mathrm{~L}_{\mathrm{a} 1} \pm \mathrm{L}_{\mathrm{a} 2} \cos \left(2 \theta_{\mathrm{r}}-\right.$ a)
$\mathrm{L}_{\mathrm{AC}}=\mathrm{L}_{\mathrm{CA}}=-1 / 2 \mathrm{~L}_{\mathrm{a} 1} \pm \mathrm{L}_{\mathrm{a} 2} \cos \left(2 \theta_{\mathrm{r}}+\alpha\right)$
$\mathrm{L}_{\mathrm{BC}}=\mathrm{L}_{\mathrm{CB}}=-1 / 2 \mathrm{~L}_{\mathrm{a} 1} \pm \mathrm{L}_{\mathrm{a} 2} \cos 2 \theta_{\mathrm{r}}, \mathrm{L}_{\mathrm{BB}}=1 / 2 \mathrm{~L}_{\mathrm{a} 1} \pm \mathrm{L}_{\mathrm{a} 2} \cos (2$ $\theta_{\mathrm{r}}-\alpha$ )
$\mathrm{L}_{\mathrm{CC}}=1 / 2 \mathrm{~L}_{\mathrm{a} 1} \pm \mathrm{L}_{\mathrm{a} 2} \cos \left(2 \theta_{\mathrm{r}}+\alpha\right), \mathrm{L}_{\mathrm{aa}}=1 / 2 \mathrm{~L}_{\mathrm{a} 1} \pm \mathrm{L}_{\mathrm{a} 2} \cos 2 \theta_{\mathrm{r}}$
$\mathrm{L}_{\mathrm{ab}}=\mathrm{L}_{\mathrm{ba}}=1 / 2 \mathrm{~L}_{\mathrm{a} 1} \pm \mathrm{L}_{\mathrm{b} 2} \cos \left(2 \theta_{\mathrm{r}}-\alpha\right), \mathrm{L}_{\mathrm{bc}}=\mathrm{L}_{\mathrm{cb}}=1 / 2 \mathrm{~L}_{\mathrm{b} 1} \pm$ $\mathrm{L}_{\mathrm{b} 2} \cos 2 \theta_{\mathrm{r}}$
$\mathrm{L}_{\mathrm{bb}}=1 / 2 \mathrm{~L}_{\mathrm{b} 1} \pm \mathrm{L}_{\mathrm{b} 2} \cos \left(2 \theta_{\mathrm{r}}-\alpha\right), \mathrm{L}_{\mathrm{cc}}=1 / 2 \mathrm{~L}_{\mathrm{a} 1} \pm \mathrm{L}_{\mathrm{b} 2} \cos \left(2 \theta_{\mathrm{r}}\right.$ $+\alpha)$
$\mathrm{L}_{\mathrm{Aa}}=\mathrm{L}_{\mathrm{aA}}=1 / 2 \mathrm{~L}_{\mathrm{b} 12} \pm \mathrm{L}_{\mathrm{b} 12} \cos 2 \theta_{\mathrm{r}}, \mathrm{L}_{\mathrm{Ab}}=\mathrm{L}_{\mathrm{bA}}=1 / 2 \mathrm{~L}_{\mathrm{a} 12} \cos \alpha$ $\pm \mathrm{L}_{\mathrm{b} 12} \cos \left(2 \theta_{\mathrm{r}}-\alpha\right)$
$\mathrm{L}_{\mathrm{Ac}}=\mathrm{L}_{\mathrm{cA}}=1 / 2 \mathrm{~L}_{\mathrm{a} 12} \cos \alpha \pm \mathrm{L}_{\mathrm{b} 12} \cos \left(2 \theta_{\mathrm{r}}+\alpha\right), \mathrm{L}_{\mathrm{Ba}}=\mathrm{L}_{\mathrm{ab}}=1 / 2 \mathrm{~L}_{\mathrm{a} 12} \cos$
$\alpha \pm \mathrm{L}_{\mathrm{b} 12} \cos \left(2 \theta_{\mathrm{r}}-\alpha\right)$
$\mathrm{L}_{\mathrm{Bb}}=\mathrm{L}_{\mathrm{bB}}=1 / 2 \mathrm{~L}_{\mathrm{a} 12} \cos \alpha \pm \mathrm{L}_{\mathrm{b} 12} \cos \left(2 \theta_{\mathrm{r}}+\alpha\right), \mathrm{L}_{\mathrm{Bc}}=\mathrm{L}_{\mathrm{cB}}=1 / 2 \mathrm{~L}_{\mathrm{a} 12}$
$\cos \alpha \pm \mathrm{L}_{\mathrm{b} 12} \cos 2 \theta_{\mathrm{r}}$
$\mathrm{L}_{\mathrm{Ca}}=\mathrm{L}_{\mathrm{aC}}=1 / 2 \mathrm{~L}_{\mathrm{a} 12} \cos \alpha \pm \mathrm{L}_{\mathrm{b} 12} \cos \left(2 \theta_{\mathrm{r}}+\alpha\right), \mathrm{L}_{\mathrm{Cb}}=\mathrm{L}_{\mathrm{bC}}=1 / 2 \mathrm{~L}_{\mathrm{a} 12}$
$\cos \alpha \pm \mathrm{L}_{\mathrm{b} 12} \cos 2 \theta_{\mathrm{r}}$
$\mathrm{L}_{\mathrm{Cc}}=\mathrm{L}_{\mathrm{cC}}=1 / 2 \mathrm{~L}_{\mathrm{a} 12} \cos \alpha \pm \mathrm{L}_{\mathrm{b} 12} \cos \left(2 \theta_{\mathrm{r}}-\alpha\right)$,
Where, $\alpha=\frac{2 \pi}{3}$, and; $\mathrm{L}_{\mathrm{a} 11}=\mathrm{L}_{\mathrm{a} 22}=\mathrm{L}_{\mathrm{a} 12}=1 / 2\left(\mathrm{~L}_{\mathrm{md}}+\mathrm{L}_{\mathrm{mq}}\right)$

$$
\mathrm{L}_{\mathrm{b} 11}=\mathrm{L}_{\mathrm{b} 22}=\mathrm{L}_{\mathrm{b} 12}=1 / 2\left(\mathrm{~L}_{\mathrm{md}}-\mathrm{L}_{\mathrm{mq}}\right)
$$

However, the expressions for the individual inductances above, can further be used for the inductance matrix for the main windings for both machines A and B.
For machine A , the inductance matrix for the main winding is (Chem-mum O, 1997)

## IV. Rotor Winding Inductance

The stages of transformation of the voltage equations are to first transform the a-b-c phase variables into q-d-o frame where the quantities are in stationary reference frame i.e. $\mathbf{d}_{r}$ and $\mathbf{q}_{\mathrm{r}}$. Since the rotor of this machine is salient pole, the axis of the rotor quantities are already in the $\mathbf{q}$ and $\mathbf{d}$ axis, so that the $\mathbf{q - d}-\mathbf{o}$ transformation need only be applied to the stator quantities.

## V. The Machine Model In Arbitrary Q-D-O Reference Frame

In order to remove the rotor position dependence on the inductances seen in (8), the voltage equations in (4) need to be transferred to q-d-o reference frame. The technique is to transform all the stator variable to an arbitrary reference frame.
Here, all the stator variable will be transform to the rotor. In the voltage equations for the main and auxiliary windings of the transfer field machine of (4), there is no need to include the rotor equation here since our intension is to adopt rotor reference frame.
Hence, the voltage equations of the main winding of the machine will after the transformation yield;
$\mathrm{V}_{\mathrm{Q}}=\omega \lambda_{\mathrm{D}}+\rho \lambda_{\mathrm{Q}}+\mathrm{ri}_{\mathrm{Q}}$
$V_{D}=\omega \lambda_{\mathrm{Q}}+\rho \lambda_{\mathrm{D}}+\mathrm{ri}_{\mathrm{D}}$
$\mathrm{V}_{\mathrm{O}}=\rho \lambda_{\mathrm{O}}+$ rio
Doing like - wise for the auxiliary and cage (rotor) windings, we have,
$\mathrm{V}_{\mathrm{q}}=\left(\omega-2 \omega_{\mathrm{r}}\right) \lambda_{\mathrm{d}}+\rho \lambda_{\mathrm{q}}+\mathrm{ri}_{\mathrm{q}}$
$\mathrm{V}_{\mathrm{d}}=\left(\omega-2 \omega_{\mathrm{r}}\right) \lambda_{\mathrm{q}}+\rho \lambda_{\mathrm{d}}+\mathrm{ri}_{\mathrm{d}}$
$V_{0}=\rho \lambda_{0}+r i_{0}$
$V_{\hat{q} r}=\left(\omega-2 \omega_{\mathrm{r}}\right) \lambda_{d r}+\rho_{\lambda \dot{q} r}+r_{\hat{q} r} i_{\hat{q} r}$
$V_{d r}=\left(\omega-2 \omega_{\mathrm{r}}\right) \lambda_{\dot{a} r}+\rho_{\lambda \dot{d} r}+r_{d r} i_{d r}$

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$L_{11}^{A}=\left[\begin{array}{ccc}L_{l s}+L_{a}-L_{m s} \cos 2 \theta_{r} & -\frac{1}{2} L_{o}-L_{m s} \cos 2\left(\theta_{r}-\frac{\pi}{3}\right) & -\frac{1}{2} L_{o}-L_{m s} \cos 2\left(\theta_{r}+\frac{\pi}{3}\right) \\ -\frac{1}{2} L_{o}-L_{m s} \cos 2\left(\theta_{r}-\frac{\pi}{3}\right) & L_{l s}+L_{a}-L_{m s} \cos 2\left(\theta_{r}+\frac{2 \pi}{3}\right) & -\frac{1}{2} L_{o}-L_{m s} \cos 2\left(\theta_{r}-\pi\right) \\ -\frac{1}{2} L_{o}+L_{m s} \cos 2\left(\theta_{r}+\frac{\pi}{3}\right) & -\frac{1}{2} L_{o}+L_{m s} \cos 2\left(\theta_{r}-\pi\right) & L_{l s}+L_{a}-L_{m s} \cos 2\left(\theta_{r}-\frac{2 \pi}{3}\right)\end{array}\right]$
For machine B , the inductance matrix for the main winding is;
$L_{11}^{B}=\left[\begin{array}{ccc}L_{l s}+L_{a}+L_{m s} \cos 2 \theta_{r} & -\frac{1}{2} L_{o}+L_{m s} \cos 2\left(\theta_{r}-\frac{\pi}{3}\right) & -\frac{1}{2} L_{o}+L_{m s} \cos 2\left(\theta_{r}+\frac{\pi}{3}\right) \\ -\frac{1}{2} L_{o}+L_{m s} \cos 2\left(\theta_{r}-\frac{\pi}{3}\right) & L_{l s}+L_{a}+L_{m s} \cos 2\left(\theta_{r}+\frac{2 \pi}{3}\right) & -\frac{1}{2} L_{o}+L_{m s} \cos 2\left(\theta_{r}-\pi\right) \\ -\frac{1}{2} L_{o}+L_{m s} \cos 2\left(\theta_{r}+\frac{\pi}{3}\right) & L_{o}+L_{m s} \cos 2\left(\theta_{r}-\pi\right) & L_{l s}+L_{a}+L_{m s} \cos 2\left(\theta_{r}-\frac{2 \pi}{3}\right)\end{array}\right]$
Hence $\mathrm{L}_{\mathrm{GG}}=L_{11}^{A}+L_{11}^{B}\left[\begin{array}{ccc}2 L_{L S}+2 L_{o} & -L_{0} & -L_{o} \\ -L_{O} & 2 L_{L S}+2 L_{O} & -L_{o} \\ -L_{0} & -L_{0} & 2 L_{L S}+2 L_{0}\end{array}\right]$
Where $\mathrm{L}_{\mathrm{Ls}}=$ leakage inductance, and $\mathrm{L}_{\mathrm{o}}=\frac{L_{m d}+L_{m q}}{2}$
$\therefore \mathrm{L}_{\mathrm{GG}}=\left[\begin{array}{ccc}2 L_{L S}+L_{m d}+L_{m q} & -\frac{1}{2}\left(L_{m d}+L_{m q}\right) & -\frac{1}{2}\left(L_{m d}+L_{m q}\right) \\ -\frac{1}{2}\left(L_{m d}+L_{m q}\right) & 2 L_{L S}+L_{m d}+L_{m q} & -\frac{1}{2}\left(L_{m d}+L_{m q}\right) \\ -\frac{1}{2}\left(L_{m d}+L_{m q}\right) & -\frac{1}{2}\left(L_{m d}+L_{m q}\right) & 2 L_{L S}+L_{m d}+L_{m q}\end{array}\right.$
Now for mutual inductance, For machine A, the mutual inductance matrix is given as;
$L_{12}^{A}=\left[\begin{array}{ccc}L_{l s}+L_{a}-L_{m s} \cos 2 \theta_{r} & L_{o}-L_{m s} \cos \left(2 \theta_{r}-\alpha\right) & L_{o} \cos \alpha-L_{m s} \cos \left(2 \theta_{r}+\alpha\right) \\ L_{o} \cos \alpha-L_{m s} \cos \left(2 \theta_{r}-\alpha\right) & L_{l s}+L_{a}-L_{m s} \cos \left(2 \theta_{r}+\alpha\right) & L_{o} \cos \alpha-L_{m s} \cos 2 \theta_{r} \\ L_{n} \cos \alpha-L_{m s} \cos \left(2 \theta_{r}+\alpha\right) & L_{n} \cos \alpha-L_{m s} \cos 2 \theta_{r} & L_{l s}+L_{a}-L_{m s} \cos \left(2 \theta_{r}-\alpha\right)\end{array}\right]$
Likewise, for machine B , the mutual inductance matrix is given as;
$L_{12}^{B}=\left[\begin{array}{lll}L_{l s}+L_{a}+L_{m s} \cos 2 \theta_{r} & L_{o}+L_{m s} \cos \left(2 \theta_{r}-\alpha\right) & L_{o} \cos \alpha+L_{m s} \cos \left(2 \theta_{r}+\alpha\right) \\ L_{o} \cos \alpha+L_{m s} \cos \left(2 \theta_{r}-\alpha\right) & L_{l s}+L_{a}+L_{m s} \cos \left(2 \theta_{r}+\alpha\right) & L_{o} \cos \alpha+L_{m s} \cos 2 \theta_{r} \\ L_{n} \cos \alpha+L_{m s} \cos \left(2 \theta_{r}+\alpha\right) & L_{n} \cos \alpha+L_{m s} \cos 2 \theta_{r} & L_{l s}+L_{a}+L_{m s} \cos \left(2 \theta_{r}-\alpha\right)\end{array}\right]$
But $\mathrm{L}_{\mathrm{GH}}=L_{12}^{A}+L_{12}^{B}$
$\left[\begin{array}{ccc}L_{G H}= & -2 L_{m s} \cos \left(2 \theta_{r}-\alpha\right) & -2 L_{m s} \cos \left(2 \theta_{r}+\alpha\right) \\ -2 L_{m s} \cos 2 \theta_{r} & & \\ -2 L_{m s} \cos \left(2 \theta_{r}-\alpha\right) & -2 L_{m s} \cos \left(2 \theta_{r}+\alpha\right) & -2 L_{m s} \cos 2 \theta_{r} \\ -2 L_{m s} \cos \left(2 \theta_{r}+\alpha\right) & -2 L_{m s} \cos 2 \theta_{r} & -2 L_{l s} \cos \left(2 \theta_{r}-\alpha\right)\end{array}\right]$
$L_{G H}=$
$\Rightarrow \quad-2 L_{m s}\left[\begin{array}{ccc}\cos 2 \theta_{r} & \cos \left(2 \theta_{r}-\alpha\right) & \cos \left(2 \theta_{r}+\alpha\right) \\ \cos \left(2 \theta_{r}-\alpha\right) & \cos \left(2 \theta_{r}+\alpha\right) & \cos 2 \theta_{r} \\ \cos \left(2 \theta_{r}+\alpha\right) & \cos 2 \theta_{r} & \cos \left(2 \theta_{r}-\alpha\right)\end{array}\right]$
But $L_{m s}=\frac{L_{m d}-L_{m q}}{2}$
$\therefore-2 \mathrm{~L}_{\mathrm{ms}}=\frac{2\left(L_{m q}-L_{m d}\right)}{2}$
$\therefore$
$L_{m q}-$
$L_{m d}\left[\begin{array}{lcr}\cos 2 \theta_{r} & \cos \left(2 \theta_{r}-\alpha\right) & \cos \left(2 \theta_{r}+\alpha\right) \\ \cos \left(2 \theta_{r}-\alpha\right) & \cos \left(2 \theta_{r}+\alpha\right) & \cos 2 \theta_{r} \\ \cos \left(2 \theta_{r}+\alpha\right) & \cos 2 \theta_{r} & \cos \left(2 \theta_{r}-\alpha\right)\end{array}\right]$
(8)

Where, $\alpha=\frac{2 \pi}{3}$
Since the main and auxiliary winding for machine $A$ and $B$ are identical, $L_{H G}$ and $L_{H H}$ will be the same as $L_{G H}$ and $L_{G G}$ respectively.

## VI. Transformation Of Flux Linkages

The ABC and abc subscripts denote variables and parameters associated with the main and auxiliary windings respectively. Both $\mathrm{r}_{\mathrm{ABC}}$ and $\mathrm{r}_{\mathrm{abc}}$ are diagonal matrices each with equal non zero elements. For a magnetically linear system, the flux linkages may be expressed as;
$\left[\begin{array}{l}\lambda_{A B C} \\ \lambda_{\text {ahr }}\end{array}\right]=\left[\begin{array}{ll}L_{G G} & L_{G H} \\ L_{H G} & L_{H H}\end{array}\right]\left[\begin{array}{l}i_{A B C} \\ i_{\text {ahr }}\end{array}\right] \mathrm{Wb}$ turns
Where $\mathrm{G}=$ main winding, $\mathrm{H}=$ Auxiliary winding.
To transform the above equation in respect to the cage winding, we have as follows,

The inductance matrix terms $\mathrm{L}_{\mathrm{GG}}, \mathrm{L}_{\mathrm{GH}}, \mathrm{L}_{\mathrm{HG}}$ and $\mathrm{L}_{\mathrm{HH}}$ are obtained from inductance sub-matrices $\mathrm{L}_{11}, \mathrm{~L}_{12}, \mathrm{~L}_{21}$ and $\mathrm{L}_{22}$ for machine A and B.
$\mathrm{L}_{\text {GRA }}$ is the mutual inductance matrix between main winding of machine A and rotor winding of machine A.
$\mathrm{L}_{\text {GRB }}$ is the mutual inductance matrix between main winding of machine $B$ and rotor winding of machine $B$.
$\mathrm{L}_{\text {HRA }}$ is the mutual inductance matrix between auxiliary winding of machine $A$ and rotor winding of machine $A$
$L_{\text {HRB }}$ is the mutual inductance matrix between auxiliary winding of machine $B$ and rotor winding of machine $B$.
$\mathrm{L}_{\text {RARA }}$ is the inductance matrix of rotor winding of machine A.
$\mathrm{L}_{\text {RARB }}$ is the mutual inductance matrix between the rotor winding of machine $A$ and the rotor winding of machine $B$.

## VII. Stator Winding Inductances

To reduce the mathematical complexities of equation 11, it is rewritten in $\mathbf{q - d} \mathbf{- o}$ frame as;
$\left[\begin{array}{lll}\lambda_{Q} & \lambda_{D} & \lambda_{o} \\ \lambda_{q} & \lambda_{d} & \lambda_{o}\end{array}\right]^{T}=\left[\begin{array}{lll}K_{G} & L_{G G} & K_{G}^{-1} \\ K_{H} & K_{H G} & L_{G H} \\ K_{G}^{-1} & K_{G} & L_{H H} \\ K_{H}^{-1}\end{array}\right]\left[\begin{array}{lll}i_{Q} & i_{D} & i_{0} \\ i_{q} & i_{d} & i_{o}\end{array}\right]$
(13)

Where $\mathrm{K}_{\mathrm{G}}=\frac{2}{3}\left[\begin{array}{ccc}\cos \theta & \cos (\theta-\alpha) & \cos (\theta+\alpha) \\ \sin \theta & \sin (\theta-\alpha) & \sin (\theta+\alpha) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{array}\right]$
$K_{G}^{-1}=\left[\begin{array}{llc}\cos \theta & \sin \theta & 1 \\ \cos (\theta-\alpha) & \sin (\theta-\alpha) & 1 \\ \cos (\theta+\alpha) & \sin (\theta+\alpha) & 1\end{array}\right]$
$\mathrm{K}_{\mathrm{H}}=\frac{2}{3}\left[\begin{array}{ccc}\cos \beta & \cos (\beta-\alpha) & \cos (\beta+\alpha) \\ \sin \beta & \sin (\beta-\alpha) & \sin (\beta+\alpha) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{array}\right]$
$K_{H}^{-1}=\left[\begin{array}{llc}\cos \beta & \sin \beta & 1 \\ \cos (\beta-\alpha) & \sin (\beta-\alpha) & 1 \\ \cos (\beta+\alpha) & \sin (\beta+\alpha) & 1\end{array}\right]$
Where $\beta=\theta=$ Speed of rotation of the arbitrary reference frame

$$
\theta_{\mathrm{r}}=\text { Angular rotor position }
$$

Therefore the flux linkage of equation 11 is now expressed as;
$\lambda_{\mathrm{Q}}=\left(2 \mathrm{~L}_{\mathrm{L}}+\mathrm{L}_{\mathrm{mq}}+\mathrm{L}_{\mathrm{md}}\right) \mathrm{i}_{\mathrm{Q}}-\left(\mathrm{L}_{\mathrm{md}}-\mathrm{L}_{\mathrm{mq}}\right)\left(\mathrm{i}_{\mathrm{q}+} i_{\text {ifr }}\right)$
$=2 \mathrm{~L}_{\mathrm{L}} \mathrm{i}_{\mathrm{Q}}+\mathrm{L}_{\mathrm{mq}} \mathrm{i}_{\mathrm{Q}}+\mathrm{L}_{\mathrm{md}} \mathrm{i}_{\mathrm{Q}}-\mathrm{L}_{\mathrm{md}}\left(\mathrm{i}_{\mathrm{q}}+i_{\text {arr }}\right)+\mathrm{L}_{\mathrm{mq}}\left(\mathrm{i}_{\mathrm{q}}+i_{\text {arr }}\right)$
$=2 \mathrm{~L}_{\mathrm{L}} \mathrm{i}_{\mathrm{Q}}+\mathrm{L}_{\mathrm{mq}} \mathrm{i}_{\mathrm{Q}}+\mathrm{L}_{\mathrm{md}} \mathrm{i}_{\mathrm{Q}}+\mathrm{L}_{\mathrm{md}} \mathrm{i}_{\mathrm{Q}}-\mathrm{L}_{\mathrm{md}} \mathrm{i}_{\mathrm{Q}}-\mathrm{L}_{\mathrm{md}}\left(\mathrm{i}_{\mathrm{q}}+\dot{i}_{\hat{q} r}\right)+$
$\mathrm{L}_{\mathrm{mq}}\left(\mathrm{i}_{\mathrm{q}}+\dot{i}_{\hat{q} r}\right)$
$=2 \mathrm{~L}_{\mathrm{L}} \mathrm{i}_{\mathrm{Q}}+2 \mathrm{~L}_{\mathrm{md}} \mathrm{i}_{\mathrm{Q}}+\mathrm{L}_{\mathrm{mq}} \mathrm{i}_{\mathrm{Q}}-\mathrm{L}_{\mathrm{md}} \mathrm{i}_{\mathrm{Q}}-\left(\mathrm{i}_{\mathrm{q}}+i_{\hat{q} r}\right)+\mathrm{L}_{\mathrm{mq}}\left(\mathrm{i}_{\mathrm{q}}+i_{\hat{q} r}\right)$
$=2\left(\mathrm{~L}_{\mathrm{L}}+\mathrm{L}_{\mathrm{md}}\right) \mathrm{i}_{\mathrm{Q}}+\left[\mathrm{i}_{\mathrm{Q}}\left(\mathrm{L}_{\mathrm{mq}}-\mathrm{L}_{\mathrm{md}}\right)+\left(\mathrm{i}_{\mathrm{q}}+i_{\hat{q} r}\right)\left(\mathrm{L}_{\mathrm{mq}}-\mathrm{L}_{\mathrm{md}}\right)\right]$
$=2\left(\mathrm{~L}_{\mathrm{L}}+\mathrm{L}_{\mathrm{md}}\right) \mathrm{i}_{\mathrm{Q}}+\left(\mathrm{i}_{\mathrm{Q}}+\mathrm{i}_{\mathrm{q}}+i_{\hat{q} r}\right)\left(\mathrm{L}_{\mathrm{mq}}-\mathrm{L}_{\mathrm{md}}\right)$
$\therefore \lambda_{\mathrm{Q}}=2\left(\mathrm{~L}_{\mathrm{L}}+\mathrm{L}_{\mathrm{md}}\right) \mathrm{i}_{\mathrm{Q}}+\left(\mathrm{L}_{\mathrm{mq}}-\mathrm{L}_{\mathrm{md}}\right)\left(\mathrm{i}_{\mathrm{Q}}+\mathrm{i}_{\mathrm{q}}+i_{\text {arr }}\right)$
Similarly;
$\lambda_{\mathrm{D}}=\left(2 \mathrm{~L}_{\mathrm{L}}+\mathrm{L}_{\mathrm{mq}}+\mathrm{L}_{\mathrm{md}}\right) \mathrm{i}_{\mathrm{D}}+\left(\mathrm{L}_{\mathrm{md}}-\mathrm{L}_{\mathrm{mq}}\right)\left(\mathrm{i}_{\mathrm{d}+} \dot{i}_{d r}\right)$
$=2\left(\mathrm{~L}_{\mathrm{L}}+\mathrm{L}_{\mathrm{mq}}\right) \mathrm{i}_{\mathrm{D}}+\left(\mathrm{L}_{\mathrm{md}}-\mathrm{L}_{\mathrm{mq}}\right)\left(\mathrm{i}_{\mathrm{D}}+\mathrm{i}_{\mathrm{d}}+i_{d r}\right)$
$\lambda_{\mathrm{O}}=2 \mathrm{~L}_{\mathrm{L}} \mathrm{i}_{\mathrm{O}}$
Also, $\lambda_{\mathrm{q}}=\left(2 \mathrm{~L}_{\mathrm{L}}+\mathrm{L}_{\mathrm{mq}}+\mathrm{L}_{\mathrm{md}}\right) \mathrm{i}_{\mathrm{q}}-\left(\mathrm{L}_{\mathrm{md}}-\mathrm{L}_{\mathrm{mq}}\right)\left(\mathrm{i}_{\mathrm{Q}}+\dot{i}_{\text {arr }}\right)$
$=2\left(\mathrm{~L}_{\mathrm{L}}+\mathrm{L}_{\mathrm{md}}\right) \mathrm{i}_{\mathrm{q}}+\left(\mathrm{L}_{\mathrm{mq}}-\mathrm{L}_{\mathrm{md}}\right)\left(\mathrm{i}_{\mathrm{Q}}+\mathrm{i}_{\mathrm{q}}+i_{\text {arr }}\right)$ (21) $\lambda_{\mathrm{d}}=\left(2 \mathrm{~L}_{\mathrm{L}}+\mathrm{L}_{\mathrm{mq}}+\mathrm{L}_{\mathrm{md}}\right) \mathrm{i}_{\mathrm{d}}+\left(\mathrm{L}_{\mathrm{md}}-\mathrm{L}_{\mathrm{mq}}\right)\left(\mathrm{i}_{\mathrm{D}}+i_{d r}\right)$ $=2\left(\mathrm{~L}_{\mathrm{L}}+\mathrm{L}_{\mathrm{mq}}\right) \mathrm{i}_{\mathrm{d}}+\left(\mathrm{L}_{\mathrm{md}}-\mathrm{L}_{\mathrm{mq}}\right)\left(\mathrm{i}_{\mathrm{D}}+\mathrm{i}_{\mathrm{d}}+i_{d r}\right)$ $\lambda_{0}=2 L_{L} i_{0}$
(23)

Also; $\lambda_{\hat{q} r}=\left(\mathrm{L}_{\mathrm{Lqr}}+\mathrm{L}_{\mathrm{mq}}+\mathrm{L}_{\mathrm{md}}\right) \boldsymbol{i}_{\hat{q} r}-\left(\mathrm{L}_{\mathrm{md}}-\mathrm{L}_{\mathrm{mq}}\right)\left(\mathrm{i}_{\mathrm{Q}}+\mathrm{i}_{\mathrm{q}}\right)$
$=\left(\mathrm{L}_{\mathrm{Lqr}}+2 \mathrm{~L}_{\mathrm{md}}\right) i_{\hat{q} r}+\left(\mathrm{L}_{\mathrm{mq}}-\mathrm{L}_{\mathrm{md}}\right)\left(\mathrm{i}_{\mathrm{Q}}+\mathrm{i}_{\mathrm{q}}+i_{\hat{q} r}\right)$
$\lambda_{d r}=\left(\mathrm{L}_{\mathrm{Ldr}}+\mathrm{L}_{\mathrm{mq}}+\mathrm{L}_{\mathrm{md}}\right) i_{d r}-\left(\mathrm{L}_{\mathrm{md}}-\mathrm{L}_{\mathrm{mq}}\right)\left(\mathrm{i}_{\mathrm{D}}+\mathrm{i}_{\mathrm{d}}\right)$
$=\left(\mathrm{L}_{\mathrm{Ldr}}+2 \mathrm{~L}_{\mathrm{mq}}\right) i_{d r}+\left(\mathrm{L}_{\mathrm{md}}-\mathrm{L}_{\mathrm{mq}}\right)\left(\mathrm{i}_{\mathrm{d}}+\mathrm{i}_{\mathrm{d}+}+i_{d r}\right)$
NB: Upper case letters represent the main winding parameters, while the lower case letters and the primed lower case letters represent the auxiliary winding parameters and rotor winding parameters respectively
As before (18-20) represent the flux linkages of the main winding circuit while (21-23) represent the flux linkages of the auxiliary winding circuit. Also (24-25) represent the flux linkages of the caged (rotor) winding circuit, r in (9 and 10) is the sum of the resistances of the main, auxiliary and rotor windings in both machine halves. Equations 18-25 can be put into (9 and 10) to yield;

$$
\begin{align*}
& \mathrm{V}_{\mathrm{Q}}=\omega \lambda_{\mathrm{D}}+\rho\left[2\left(\mathrm{~L}_{\mathrm{L}}+\mathrm{L}_{\mathrm{md}}\right) \mathrm{i}_{\mathrm{Q}}+\left(\mathrm{L}_{\mathrm{mq}}-\mathrm{L}_{\mathrm{md}}\right)\left(\mathrm{i}_{\mathrm{Q}}+\mathrm{i}_{\mathrm{q}}+i_{\hat{\text { qur }}}\right)\right]+\mathrm{ri}_{\mathrm{Q}} \\
& \text { (26) } \\
& \mathrm{V}_{\mathrm{q}}=\left(\omega-2 \omega_{\mathrm{r}}\right) \lambda_{\mathrm{d}}+\rho\left[2\left(\mathrm{~L}_{\mathrm{L}}+\mathrm{L}_{\mathrm{md}}\right) \mathrm{i}_{\mathrm{q}}+\left(\mathrm{L}_{\mathrm{mq}}-\mathrm{L}_{\mathrm{md}}\right)\left(\mathrm{i}_{\mathrm{Q}}+\mathrm{i}_{\mathrm{q}}+\right.\right. \\
& \left.\left.i_{\text {ar }}\right)\right]+\mathrm{ri}_{\mathrm{q}} \quad \text { (27) } \\
& V_{\hat{G} r}=\left(\omega-2 \omega_{\mathrm{r}}\right) \lambda_{d r}+\rho\left[\left(\mathrm{L}_{\mathrm{Lqr}}+2 \mathrm{~L}_{\mathrm{md}}\right) i_{\hat{a r} r}+\left(\mathrm{L}_{\mathrm{mq}}-\mathrm{L}_{\mathrm{md}}\right)\left(\mathrm{i}_{\mathrm{Q}}+\mathrm{i}_{\mathrm{q}}+\right.\right. \\
& \left.i_{\hat{a} r}\right)+r i_{\hat{a} r} \quad \text { (28) } \\
& \mathrm{V}_{\mathrm{D}}=\omega \lambda_{\mathrm{Q}}+\rho\left[2\left(\mathrm{~L}_{\mathrm{L}}+\mathrm{L}_{\mathrm{mq}}\right) \mathrm{i}_{\mathrm{D}}+\left(\mathrm{L}_{\mathrm{md}}-\mathrm{L}_{\mathrm{mq}}\right)\left(\mathrm{i}_{\mathrm{D}}+\mathrm{i}_{\mathrm{d}}+i_{d r}\right)\right]+ \\
& \text { rid }_{D} \quad \text { (29) } \\
& \mathrm{V}_{\mathrm{d}}=\left(\omega-2 \omega_{\mathrm{r}}\right) \lambda_{\mathrm{q}}+\rho\left[2\left(\mathrm{~L}_{\mathrm{L}}+\mathrm{L}_{\mathrm{mq}}\right) \mathrm{i}_{\mathrm{d}}+\left(\mathrm{L}_{\mathrm{md}}-\mathrm{L}_{\mathrm{mq}}\right)\left(\mathrm{i}_{\mathrm{D}}+\mathrm{i}_{\mathrm{d}}+\right.\right. \\
& \left.\left.i_{d r}\right)\right]+\mathrm{ri}_{\mathrm{d}}  \tag{30}\\
& V_{d r}=\left(\omega-2 \omega_{\mathrm{r}}\right) \lambda_{\dot{a} r}+\left[\rho\left(\mathrm{L}_{\mathrm{Ldr}}+2 \mathrm{~L}_{\mathrm{mq}}\right) i_{\hat{d}_{d r}}+\left(\mathrm{L}_{\mathrm{md}}-\mathrm{L}_{\mathrm{mq}}\right)\left(\mathrm{i}_{\mathrm{d}}+\mathrm{i}_{\mathrm{d}+}\right.\right. \\
& \left.\left.i_{d r}\right)\right]+r i_{d r}
\end{align*}
$$

Also for O-variables ;

$$
\begin{align*}
\mathrm{V}_{\mathrm{O}} & =\rho \lambda_{0}+\mathrm{ri}_{0} \\
& =\rho\left(2 \mathrm{~L}_{\mathrm{L}} \mathrm{i}_{O}\right)+\text { rio }  \tag{32}\\
\mathrm{V}_{\mathrm{O}} & =\rho \lambda_{0}+\mathrm{ri}_{0} \tag{33}
\end{align*}
$$

$=\rho\left(2 L_{L} i_{o}\right)+r i_{o}$

$$
\begin{align*}
V_{\text {orr }} & =\rho \lambda_{\text {ór }}+\mathrm{ri} \dot{i}_{\text {orr }} \\
& =\rho\left(\mathrm{L}_{\mathrm{r}} \dot{i}_{\text {orr }}\right)+\dot{i}_{\text {orr }} \tag{34}
\end{align*}
$$



Fig 3- Arbitrary reference frame equivalent circuit for a 3-phase symmetrical transfer field machine with cage (rotor) winding in the $q$-variable.

Also, equations 29-31 result the equivalent circuit shown in


Figure 4: Arbitrary reference frame equivalent circuit for a 3-phase symmetrical transfer field machine with cage (rotor) winding in the d-variable.
Similarly, equations 32-34 combine to yield the equivalent circuit shown in figure 5.


Figure 5: Arbitrary reference frame equivalent circuit for a 3-phase symmetrical transfer field machine with cage (rotor) winding in the O -variable

## VIII. Rotor To Stator Winding Inductances

Obviously, both rotors of the machine halves are identical. Therefore, they possess equal and similar parameters. Let us consider the inductances between the rotor winding, and the stator windings of machine A . The winding placements are depicted in figure 6 below

From figure 6,
$\mathrm{L}_{\mathrm{GRA}}=\mathrm{L}_{\mathrm{RAG}}=\left[\begin{array}{l}L_{A q} L_{A d} \\ L_{B q} \\ L_{B d} \\ L_{C q} \\ L_{C d}\end{array}\right]$
$\mathrm{L}_{\mathrm{HRB}}=\mathrm{L}_{\mathrm{RBH}}=\left[\begin{array}{ll}L_{a q} & L_{a d} \\ L_{b q} & L_{b d} \\ L_{c q} & L_{c d}\end{array}\right]$
NB $L_{G R A}=L_{\text {RRB }}$ on the account if the identity of the two machine halves.

Also
$\mathrm{L}_{\mathrm{aq}}=\mathrm{L}_{\mathrm{Aq}}=\mathrm{L}_{\mathrm{mq}} \cos \theta \mathrm{r}, \mathrm{L}_{\mathrm{ad}}=\mathrm{L}_{\mathrm{Ad}}=\mathrm{L}_{\mathrm{md}} \sin \theta \mathrm{r}$
$L_{b q}=L_{B q}=L_{m q} \cos \left(\theta r-\frac{2 \pi}{3}\right), L_{b d}=L_{B d}=L_{m d} \sin \left(\theta r-\frac{2 \pi}{3}\right)$
$\mathrm{L}_{\mathrm{cq}}=\mathrm{L}_{\mathrm{Cq}}=\mathrm{L}_{\mathrm{mq}} \cos \left(\theta \mathrm{r}-\frac{4 \pi}{3}\right), \mathrm{L}_{\mathrm{cd}}=\mathrm{L}_{\mathrm{Cd}}=\mathrm{L}_{\mathrm{md}} \sin \left(\theta \mathrm{r}-\frac{4 \pi}{3}\right)$
(36)

## IX. Rotor To Rotor Winding Inductances

On the account of identity of the two machine halves;
$\mathrm{L}_{\text {RARA }}=\mathrm{L}_{\text {RBRB }}=\left[\begin{array}{ccc} & L_{l d r}+L_{m d} & 0 \\ 0 & & L_{l d r}+L_{m d}\end{array}\right]$

## X. The Torque Equation Of The 3-Phase Transfer Field Machine With Cage (Rotor) Winding

The torque equation of the configured machine is obtained by integrating the rotor winding parameters into the derived torque equation of the conventional 3 -phase transfer field machine with no rotor winding.
The expression for the torque equation of the cage winding transfer field machine is given as;

$$
\begin{aligned}
& \mathrm{T}_{\mathrm{e}}=\frac{3}{2}\left(\frac{P}{2}\right) {\left[\left(i_{Q s}+i_{q s}\right) X_{m q}\left(i_{D s}+i_{d s}+i_{d s}\right]\right.} \\
&-\left[\left(i_{D s}+i_{d s}\right) X_{m q}\left(i_{Q s}+i_{q s}+i_{\hat{q} s}\right)\right](38)
\end{aligned}
$$

Where,
$\mathrm{i}_{\mathrm{Q} s}$ is the q -axis stator current in the main winding of transfer field machine
$\mathrm{i}_{\mathrm{qs}}$ is the q -axis stator current in the auxiliary winding of transfer field machine
$\mathrm{i}_{\mathrm{Ds}}$ is the d -axis stator current in the main winding of transfer field machine
$\mathrm{i}_{\mathrm{ds}}$ is the d-axis stator current in the auxiliary winding of transfer field machine idr is the d axis rotor current in the (cage) rotor of transfer field machine.
$\dot{i}_{\hat{a} r}$ is the q-axis rotor current in the (cage) rotor of transfer field machine


Figure 6: Rotor to Stator winding inductances

M/C A

XI. Circuit Parameters For The Dynamic State Simulation Of The Machine

Table 1

| S/No | Parameter | Value |
| :--- | :--- | :--- |
| 1 | $\mathrm{~L}_{\mathrm{md}}$ | 133.3 mH |
| 2 | $\mathrm{~L}_{\mathrm{mq}}$ | 25.6 mH |
| 3 | $\mathrm{~L}_{\mathrm{Ls}}=\mathrm{L}_{\mathrm{ia}}=\mathrm{L}_{\mathrm{er}}$ | 0.6 mH |
| 4 | $\mathrm{r}_{\mathrm{m}}=\mathrm{r}_{\mathrm{a}}=\mathrm{r}_{\mathrm{r}}=\mathrm{r}$ | $3.0 \Omega$ |
| 5 | J | $1.98 \times 10^{-3} \mathrm{kgm}^{3}$ |
| 6 | V | 220 V |
| 7 | F | $50 \mathrm{H}_{\mathrm{Z}}$ |
| 8 | P | 2 |

Matlab plots for the machine output characteristics using (1) through (38), the plots of the machine's output characteristics are shown in figure 7 and 8.


Figure 7: Rotor speed run up plot for the configured machine


Figure 8: A Plot of Electromagnetic magnetic torque verses Time

## XII. Result And Analysis Of The Machine

For the dynamic operation of the machine, the rotor speed run-up plot against time for the configured (cage) machine is shown in figure 7. There was a little transient at different stages while rotor speed builds up before an application of load at 7 seconds. After another little transient, the rotor speed now settles to a steady-state at about $1410 \mathrm{~N}-\mathrm{m}$. Also the graph of electromagnetic torque against time for the caged machine with oscillations noticed at different stages is shown in figure 8. It is observed that on no-load, value for electromagnetic torque is zero. On application of load torque at 6.9 seconds to the machine, it oscillates and settled to a steady-state of 3.4 N .

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## XIII. Conclusion

From the analysis of the improved transfer field machine, the inclusion of rotor winding into the conventional machine provides better output performance characteristics, necessary for its wider applications in engineering industries (Fitzgerald A.E, Charles K. Jr, Stephen D.U. 2003), (Gupta J.B 2006), (Menta, V.K. Rotit Menta (2000).

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