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Dynamics of Two Fluid Bianchi Type-III Cosmological Models with Constant Deceleration Parameter

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Abstract— In this present work we have investigated dynamics of the Bianchi type-III universe filled with dark and barotropic fluids. Here we have analyze both interacting and non-interacting fluids. The law of variation of Hubble parameter which gives constant value of deceleration parameter is applied in order to obtain the solutions of the field equations. Cosmological parameters of the models are obtained and studied. In this present work we have investigated dynamics of the Bianchi type-III universe filled with dark and barotropic fluids. Here we have analyze both interacting and non-interacting fluids. The law of variation of Hubble parameter which gives constant value of deceleration parameter is applied in order to obtain the solutions of the field equations. Cosmological parameters of the models are obtained and studied.

Index Terms— Two fluids, Bianchi type-III space-time, Constant Deceleration Parameter

I. INTRODUCTION

As Bianchi type models are homogeneous and anisotropic, it helps to study the early anisotropy of the universe and late time isotropization. Bianchi type models are generalization of FRW (isotropic) model. The ground and altitudinal experiments done in recent years proved that the universe is spatially flat on large scale whereas it is expanding with acceleration [1-14]. To derive this acceralation there must exist some exotic form of energy called Dark energy (DE) having negative pressure. The Dark energy density parameter is $\Omega_{DE} \approx 0.70$

To solve Einstein's field equations most of the times we assume the law of variation of Hubble's parameter [15]. This law results into a constant value of Deceleration Parameter (DP). This law gives two types of models, one gives exponential volumetric expansion model and other is power-law volumetric expansion model having singular (Big-Bang) and non-singular origin of the universe respectively. Most of the Einstein's theory models and Branse-Dicke models with curvature parameter are constant DP models.

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Dark energy models are used in various aspects like higher derivative terms model [16], two field dilation model [17], dark energy model with viscosity, variable and G [18], modified chaplygin gas with dark matter and holographic dark energy [19], the interacting and non-interacting tachyon [20]. Two dark fluid model with variavle DP [21 - 23].The dimensionless jerk parameter j is studied in order to get our model close to CDM [24-28]. .

II. METRIC AND BASIC FIELD EQUATIONS

The spatially homogeneous and anisotropic Bianchi type-III model can be written as

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2\alpha x} dy^2 - C^2 \ dz^2 \quad , \qquad (1)$$

where $\alpha \neq 0$ is a constant and A, B, C are scale factors and are functions of cosmic time 't' .

The Einstein field equations are (For $8\pi G = 1, c = 1$)

$$R_i^j - \frac{1}{2}\delta_i^j R = -(T_i^{j(B)} + T_i^{j(D)}) , \qquad (2)$$

where R_i^j is the Ricci tensor, R is the Ricci scalar and $T_i^{j(B)}$ and $T_i^{j(D)}$ are the energy momentum tensor for barotropic fluid and dark fluid respectively.

The energy momentum tensors are

$$T_i^{j(B)} = (\rho_B + p_B) u_i u^j - p_B ,$$
$$T_i^{j(D)} = (\rho_D + p_D) u_i u^j - p_D ;$$

with components

$$T_1^{1(B)} = T_2^{2(B)} = T_3^{3(B)} = -p_B \quad , \quad T_4^{4(B)} = \rho_B ,$$

$$T_1^{1(D)} = T_2^{2(D)} = T_3^{3(D)} = -p_D \quad , \quad T_4^{4(D)} = \rho_D , \qquad (3)$$

where p_B , ρ_B and p_D , ρ_D are pressures and densities of barotropic fluid and dark fluid respectively which satisfy the barotropic equations of state

$$p_B = w_B \rho_B ,$$
$$p_D = w_D \rho_D . \qquad (4)$$

The Einstein's field equations (2) for Bianchi type-III line element (1) using equations (2) and (3) are

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$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=-(p_B+p_D) \quad , \qquad (5)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=-(p_B+p_D) \quad , \qquad (6)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\alpha^2}{A^2}=-(p_B+p_D), \qquad (7)$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{\alpha^2}{A^2}=\rho_B+\rho_D, \qquad (8)$$

$$\frac{\dot{B}}{B}-\frac{\dot{A}}{A}=0, \qquad (9)$$

where overhead dots denotes derivatives with respect to cosmic time 't'.

The spatial volume V is given by

$$V^3=ABC. \qquad (10)$$

We define mean scale factor 'a' as

$$a=(ABC)^{\frac{1}{3}}. \qquad (11)$$

The mean Hubble parameter is defined by

$$H=\frac{\dot{a}}{a}=\frac{1}{3}\left[\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right]. \qquad (12)$$

The deceleration parameter q which is an important observational quantity is defined as

$$q=-\frac{\ddot{a}\,a}{\dot{a}^2}. \qquad (13)$$

III. SOLUTIONS OF THE FIELD EQUATIONS

On integrating equation (9), we obtain

$$B=\mu A,$$

where μ is a constant of integration and for $\mu=1$ above equation reduces to

$$B=A. \qquad (14)$$

The field equations (5) to (8) with the help of equation (14) reduces to

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=-(p_B+p_D), \qquad (15)$$

$$2\frac{\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}-\frac{\alpha^2}{A^2}=-(p_B+p_D), \qquad (16)$$

$$\frac{\dot{A}^2}{A^2}+2\frac{\dot{A}\dot{C}}{AC}-\frac{\alpha^2}{A^2}=\rho_B+\rho_D \quad . \qquad (17)$$

Equations (15), (16) and (17) gives

$$p_{tot}=-\frac{1}{2}\left[3\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{A}^2}{A^2}-\frac{\alpha^2}{A^2}\right] \qquad (18)$$

$$\rho_{tot}=\frac{\dot{A}^2}{A^2}+2\frac{\dot{A}\dot{C}}{AC}-\frac{\alpha^2}{A^2}, \qquad (19)$$

where $p_{tot}=p_B+p_D$ and $\rho_{tot}=\rho_B+\rho_D$.

The constant deceleration parameter q, is obtained by applying variation law for generalized Hubble parameter H as

$$H=l\,a^{-n}=l\,(ABC)^{\frac{-n}{3}}, \qquad (20)$$

where $n\geq 0$ and $l>0$ are constants.

Equations (12) and (20) gives us

$$\dot{a}=l\,a^{-n+1} \qquad (21)$$

and

$$\ddot{a}=-l^2(n-1)a^{-2n+1}. \qquad (22)$$

Using equations (21) and (22) in the equation (13), we get the constant DP as

$$q=n-1 \quad . \qquad (23)$$

For $n>1$, the DP q is positive indicating decelerating model whereas for $0\leq n<1$ indicates accelerating model. The law of average scale factor 'a' can be obtained by integrating (21) as

$$a=(n\,lt+c_1)^{\frac{1}{n}} \quad \text{for } n\neq 0 \qquad (24)$$

$$a=c_2e^{lt} \quad \text{for } n=0, \qquad (25)$$

where c_1 and c_2 are constants of integration.

Therefore this law gives us two types of the expansion in the universe. First is power-law volumetric expansion and another is exponential volumetric expansion model.

For solving above equations we use the condition

$$C=A^k, \qquad (26)$$

we get the total pressure and total density as

$$p_{tot}=-\frac{1}{2}\left[(3+k)\frac{\ddot{A}}{A}+(k^2+1)\frac{\dot{A}^2}{A^2}-\frac{\alpha^2}{A^2}\right], \qquad (27)$$

$$\rho_{tot}=(1+2k)\frac{\dot{A}^2}{A^2}-\frac{\alpha^2}{A^2}. \qquad (28)$$

The Bianchi identity $G_{ij}^{;j}=0$ implies that $T_{ij}^{;j}=0$ which gives

$$\dot{\rho}_{tot}+(2+k)\frac{\dot{A}}{A}(p_{tot}+\rho_{tot})=0. \qquad (29)$$

IV. NON-INTERACTING CASE

Here we assume that there is no interaction between two fluids So the separate conservation equations for barotropic fluid and dark fluid are considered (∵ the interacting term $Q=0$).

(18)

$$\dot{\rho}_B+(2+k)\frac{\dot{A}}{A}(p_B+\rho_B)=0, \qquad (30)$$

(19)

$$\dot{\rho}_D+(2+k)\frac{\dot{A}}{A}(p_D+\rho_D)=0 \quad . \qquad (31)$$

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Solving (30), we get

$$\rho_B = \rho_0 \, A^{-(k+2)(1+w_B)} \quad , \tag{32}$$

where ρ_0 is a constant of integration.

Using equations (28) and (27), the density and pressure of dark fluid are given by

$$\rho_D = (1+2k)\frac{\dot{A}^2}{A^2} - \frac{\alpha^2}{A^2} - \rho_0 \, A^{-(k+2)(1+w_B)} \quad , \tag{33}$$

$$p_D = -\frac{1}{2}\left[(3+k)\frac{\ddot{A}}{A} + (k^2+1)\frac{\dot{A}^2}{A^2} - \frac{\alpha^2}{A^2}\right] - w_B \, \rho_0 \, A^{-(k+2)(1+w_B)} \quad . \tag{34}$$

Now we consider two separate cases.

Case I : Power -Law Expansion model {For $n \neq 0$}

Consider the scale factor using equations (20) and (24) as

$$A(t) = (nlt + c_1)^{\frac{3}{n(k+2)}} \quad , \tag{35}$$

where $n > 0$ is a constant.

Using equation (35), the density and pressure for barotropic fluid is

$$\rho_B = \rho_0 \, (nlt + c_1)^{\frac{-3(1+w_B)}{n}} \quad , \tag{36}$$

$$p_B = \rho_0 \, w_B (nlt + c_1)^{\frac{-3(1+w_B)}{n}} \quad . \tag{37}$$

Using (35) in (33) and (34) we get density and pressure for dark fluid as

$$\rho_D = \frac{9(1+2k)l^2}{(k+2)^2 (nlt+c_1)^2} - \frac{\alpha^2}{(nlt+c_1)^{\frac{6}{n(k+2)}}} - \rho_0 \, (nlt+c_1)^{\frac{-3(1+w_B)}{n}} \tag{38}$$

$$p_D = -\frac{1}{2}\left[\frac{3l^2[(3+k)(3-n(k+2)) + 3(k^2+1)]}{(k+2)^2 (nlt+c_1)^2} - \frac{\alpha^2}{(nlt+c_1)^{\frac{6}{n(k+2)}}}\right] - w_B \, \rho_0 \, (nlt+c_1)^{\frac{-3(1+w_B)}{n}} . \tag{39}$$

The equation of state (EoS) parameter for dark energy is

$$w_D = \frac{p_D}{\rho_D} = -\frac{\frac{1}{2}\left[\frac{3l^2[(3+k)(3-n(k+2)) + 3(k^2+1)]}{(k+2)^2 (nlt+c_1)^2} - \frac{\alpha^2}{(nlt+c_1)^{\frac{6}{n(k+2)}}}\right] - w_B \, \rho_0 \, (nlt+c_1)^{\frac{-3(1+w_B)}{n}}}{\frac{9(1+2k)l^2}{(k+2)^2 (nlt+c_1)^2} - \frac{\alpha^2}{(nlt+c_1)^{\frac{6}{n(k+2)}}} - \rho_0 \, (nlt+c_1)^{\frac{-3(1+w_B)}{n}}} . \tag{40}$$

The expression for density parameter of Barotropic and dark energy densities and total energy density are obtained as

$$\Omega_B = \frac{\rho_B}{3H^2} = \frac{\rho_0}{3l^2} \, (nlt + c_1)^{-\frac{3(1+w_B)}{n} + 2} \quad , \tag{41}$$

$$\Omega_D = \frac{\rho_D}{3H^2} = \frac{1}{3l^2}\left[\frac{9(1+2k)l^2}{(k+2)^2} - \frac{\alpha^2}{(nlt+c_1)^{\frac{6}{n(k+2)}-2}} - \rho_0 \, (nlt+c_1)^{\frac{-3(1+w_B)}{n}+2}\right] \tag{42}$$

$$\Omega = \Omega_B + \Omega_D = \frac{3(1+2k)}{(2+k)^2} - \frac{\alpha^2}{3l^2 \, (nlt+c_1)^{\frac{6}{n(k+2)}-2}} . \tag{43}$$

Case II : Exponential volumetric Expansion Model {For $n = 0$}:

Here, the scale factor is defined as

$$A(t) = (c_2 e^{lt})^{\frac{3}{k+2}} \quad . \tag{44}$$

By using scale factor (44), density and pressure for barotropic fluid is

$$\rho_B = \rho_0 \, (c_2 \, e^{lt})^{-3(1+w_B)} \tag{45}$$

$$p_B = \rho_0 \, w_B \, (c_2 \, e^{lt})^{-3(1+w_B)} . \tag{46}$$

Similarly equations (34) and (35) gives density and pressure for dark fluid as

$$\rho_D = \frac{9(1+2k)l^2}{(k+2)^2} - \frac{\alpha^2}{(c_2 e^{lt})^{\frac{6}{k+2}}} - \rho_0 \, (c_2 \, e^{lt})^{-3(1+w_B)} \quad , \tag{47}$$

$$p_D = -\frac{9l^2 \, (k^2+k+4)}{2(k+2)^2} + \frac{\alpha^2}{2(c_2 e^{lt})^{\frac{6}{k+2}}} - w_B \, \rho_0 \, (c_2 \, e^{lt})^{-3(1+w_B)} \tag{48}$$

The EoS parameter of dark energy fluid is obtained as

$$w_D = \frac{p_D}{\rho_D} = \frac{-\frac{9l^2 \, (k^2+k+4)}{2(k+2)^2} + \frac{\alpha^2}{2(c_2 e^{lt})^{\frac{6}{k+2}}} - w_B \, \rho_0 \, (c_2 \, e^{lt})^{-3(1+w_B)}}{\frac{9(1+2k)l^2}{(k+2)^2} - \frac{\alpha^2}{(c_2 e^{lt})^{\frac{6}{k+2}}} - \rho_0 \, (c_2 \, e^{lt})^{-3(1+w_B)}} . \tag{49}$$

The expression for density parameter of barotropic and dark energy densities and total energy density are obtained as

$$\Omega_B = \frac{\rho_B}{3H^2} = \frac{\rho_0 (c_2 \, e^{lt})^{-3(1+w_B)}}{3l^2} \quad , \tag{50}$$

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$$\Omega_D=\frac{\rho_D}{3H^2}=\frac{3(1+2k)}{(k+2)^2}-\frac{\alpha^2}{3l^2(c_2e^{lt})^{\frac{6}{k+2}}}-\frac{\rho_0}{3l^2}(c_2\,e^{lt})^{-3\,(1+w_B)}, \quad (51)$$

$$\Omega=\Omega_B+\Omega_D=\frac{3(1+2k)}{(k+2)^2}-\frac{\alpha^2}{3l^2\,(c_2e^{lt})^{\frac{6}{k+2}}}\ . \quad (52)$$

V. Interacting Case

Here we consider the interaction between barotropic and dark fluid. So the conservation equations for barotropic fluid and dark fluid are as follows

$$\dot{\rho}_B+(2+k)\frac{\dot{A}}{A}(p_B+\rho_B)=Q\ , \quad (53)$$

$$\dot{\rho}_D+(2+k)\frac{\dot{A}}{A}(p_D+\rho_D)=-Q\ . \quad (54)$$

Here we consider the interacting term Q as

$$Q=3H\,\sigma\,\rho_B\ . \quad (55)$$

Where σ is a coupling constant.

On integrating equation (53) and using equation (55), we get

$$\rho_B=\rho_0\ A^{-(2+k)(1+w_B-\sigma)} \quad (56)$$

where ρ_0 is a constant of integration.

By using equation (56), we get p_B , ρ_D and p_D in terms of scale factor as

$$p_B=\rho_0\ w_B\ A^{-(2+k)(1+w_B-\sigma)}\ , \quad (57)$$

$$\rho_D=(1+2k)\frac{\dot{A}^2}{A^2}-\frac{\alpha^2}{A^2}-\rho_0\ A^{-(k+2)(1+w_B-\sigma)}\ , \quad (58)$$

$$p_D=-\frac{1}{2}\left[(3+k)\frac{\ddot{A}}{A}+(k^2+1)\frac{\dot{A}^2}{A^2}-\frac{\alpha^2}{A^2}\right]-w_B\,\rho_0\,A^{-(k+2)(1+w_B-\sigma)}\ . \quad (59)$$

Case I : Power -Law Expansion model {For $n\neq0$}

Using equation (35) in the equation (56) and (57), we get

$$\rho_B=\rho_0\,(nlt+c_1)^{-\frac{3(1+w_B-\sigma)}{n}}\ , \quad (60)$$

$$p_B=\rho_0\ w_B\,(nlt+c_1)^{-\frac{3(1+w_B-\sigma)}{n}}\ .$$

Also, the density and pressure for dark fluid using equation (58) and (59) are

$$\rho_D=\frac{9(1+2k)l^2}{(k+2)^2\,(nlt+c_1)^2}-\frac{\alpha^2}{(nlt+c_1)^{6/n(k+2)}}-\rho_0\,(nlt+c_1)^{-3(1+w_B-\sigma)/n} \quad (61)\ ,$$

$$p_D=-\frac{1}{2}\left(\frac{3l^2\,[(3+k)[3-n(k+2)]+3(k^2+1)]}{(k+2)^2\,(nlt+c_1)^2}-\frac{\alpha^2}{(nlt+c_1)^{6/n(k+2)}}\right)-w_B\rho_0\,(nlt+c_1)^{-3(1+w_B-\sigma)/n}\ . \quad (62)$$

The EoS parameter for dark fluid is

$$w_D=\frac{p_D}{\rho_D}=-\frac{-\frac{1}{2}\left(\frac{3l^2\,[(3+k)[3-n(k+2)]+3(k^2+1)]}{(k+2)^2\,(nlt+c_1)^2}-\frac{\alpha^2}{(nlt+c_1)^{6/n(k+2)}}\right)-w_B\rho_0\,(nlt+c_1)^{-3(1+w_B-\sigma)/n}}{\frac{9(1+2k)l^2}{(k+2)^2\,(nlt+c_1)^2}-\frac{\alpha^2}{(nlt+c_1)^{6/n(k+2)}}-\rho_0\,(nlt+c_1)^{-3(1+w_B-\sigma)/n}}\ . \quad (63)$$

The expression for density parameter of barotropic and dark energy densities and total energy density are obtained as

$$\Omega_B=\frac{\rho_0}{3l^2}(nlt+c_1)^{-\frac{3(1+w_B-\sigma)}{n}+2}\ , \quad (64)$$

$$\Omega_D=\frac{3(1+2k)}{(k+2)^2}-\frac{\alpha^2}{3l^2\,(nlt+c_1)^{\frac{6}{n(k+2)}-2}}-\frac{\rho_0}{3l^2}(nlt+c_1)^{\frac{-3(1+w_B-\sigma)}{n}+2}\ , \quad (65)$$

$$\Omega=\Omega_B+\Omega_D=\frac{3(1+2k)}{(k+2)^2}-\frac{\alpha^2}{3l^2\,(nlt+c_1)^{\frac{6}{n(k+2)}-2}}\ . \quad (66)$$

Case II : Exponential Volumetric Expansion model {For $n=0$}:

The expression for the density of barotropic fluid using equation (44) in the equation (56) is

$$\rho_B=\rho_0\,(c_2e^{lt})^{-3(1+w_B-\sigma)}$$

$$P_B=\rho_0\ w_B\,(c_2e^{lt})^{-3(1+w_B-\sigma)} \quad (67)$$

We obtain the density, pressure and EoS parameter of dark fluid as

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$$\rho_D = \frac{9(1+2k)l^2}{(k+2)^2} - \frac{\alpha^2}{(c_2\, e^{lt})^{\frac{6}{k+2}}} - \rho_0\,(c_2\, e^{lt})^{-3(1+w_B-\sigma)}\,, \tag{68}$$

$$p_D = -\frac{1}{2}\left(\frac{9l^2\,(k^2+k+4)}{(k+2)^2} - \frac{\alpha^2}{(c_2\, e^{lt})^{6/(k+2)}}\right) - w_B\rho_0\,(c_2\, e^{lt})^{-3(1+w_B-\sigma)}\,, \tag{69}$$

$$w_D = \frac{p_D}{\rho_D} = \frac{-\frac{1}{2}\left(\frac{9l^2\,(k^2+k+4)}{(k+2)^2} - \frac{\alpha^2}{(c_2\, e^{lt})^{6/(k+2)}}\right) - w_B\rho_0\,(c_2 e^{lt})^{-3(1+w_B-\sigma)}}{\frac{9(1+2k)l^2}{(k+2)^2} - \frac{\alpha^2}{(c_2\, e^{lt})^{6/(k+2)}} - \rho_0\,(c_2\, e^{lt})^{-3(1+w_B-\sigma)}}\,. \tag{70}$$

The expression for density parameter of barotropic and dark energy densities and total energy density are obtained as

$$\Omega_B = \frac{\rho_0}{3l^2}\,(c_2\, e^{lt})^{-3(1+w_B-\sigma)}\,,$$

$$\Omega_D = \frac{3(1+2k)}{(k+2)^2} - \frac{\alpha^2}{3l^2\,(c_2\, e^{lt})^{\frac{6}{(k+2)}}} - \frac{\rho_0}{3l^2}\,(c_2\, e^{lt})^{-3(1+w_B-\sigma)}$$

$$\Omega = \frac{3(1+2k)}{(k+2)^2} - \frac{\alpha^2}{3l^2\,(c_2\, e^{lt})^{\frac{6}{(k+2)}}}\,.$$

VI. Jerk Parameter

In cosmological jerk parameter is defined as

$$j(t) = \frac{1}{H^3}\frac{\dddot{a}}{a} = \frac{(a^2 H^2)''}{2H^2} = 2q^2 - q - \frac{\dot{q}}{H}\,.$$

$$j(t) = (1-n)(1-2n)\,.$$

VII. Conclusion

We have investigated the system of two interacting and non-interacting fluids in Bianchi type-III space-time with interacting and non-interacting barotropic and dark fluids. The solutions of the field equations are obtained by imposing the law of Hubble parameter which gives the constant value of the DP. Here we have studied two forms, the first is power-law expansion of universe (*i.e.* for $n \neq 0$) which gives the positive value of DP (for $n > 1$). The DP is negative for $0 \leq n < 1$. The second one is exponential volumetric expansion law (*i.e.* for $n = 0$) which gives negative value of DP. We observe that sum of the both barotropic and dark-energy fluid densities are the same in both interacting and non-interacting cases. So the total energy density parameters have the same properties in interacting and non-interacting models. We have obtained the value of jerk parameter $j(t) \approx 2.08$ for $n \approx 1.8$ which matches with the recent cosmological observations [29, 30, 31].

Conflict of Interest

On behalf of author states that there is no conflict of interest.

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And obtained as

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